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**ENERGY LEVELS
AND TRANSITION PROBABILITIES
OF Mn⁴⁺: TiO₂ AND Cr³⁺: ZnWO₄**

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**ENERGY LEVELS AND TRANSITION PROBABILITIES
OF $Mn^{4+} : TiO_2$ and $Cr^{3+} : ZnWO_4$**

by

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Abstract

Data on the energy levels and transition probabilities for $Mn^{4+}:TiO_2$ and $Cr^{3+}:ZnWO_4$ are presented in this report. The important parameters are the magnitude and orientation of the external d.c. magnetic field. In addition, the orientation of a linearly polarized r.f. magnetic field which results in a maximum transition probability is tabulated.

Introduction:

In order to determine the usefulness of a paramagnetic spin system in a maser it is necessary to know several characteristics of the system: (a) relaxation times, (b) energy levels, and (c) transition matrix elements. The latter two characteristics determine the operating frequencies and signal and pump transition strengths.

We present, herein, data on energy levels and transition probabilities as a function of the strength and orientation of an external d.c. magnetic field for $\text{Mn}^{4+}:\text{TiO}_2$ and $\text{Cr}^{3+}:\text{ZnWO}_4$.

Spin Hamiltonian [Andresen, 1960; Kurtz and Nilsen, 1962]

Both ions, in their respective host crystals, are surrounded by a slightly deformed octahedron of oxygen atoms. The resulting spin Hamiltonian for each is

$$(1) \quad \mathcal{H} = \beta(g_x H_x S_x + g_y H_y S_y + g_z H_z S_z) + D [S_z^2 - \frac{1}{3} S(S+1)] + E(S_x^2 - S_y^2) :$$

for

$$\text{Mn}^{4+}:\text{TiO}_2, \quad g_x = g_y = g_z = 1.99, \quad D = -12.1 \text{ GHz}, \quad E = -3.88 \text{ GHz} ;$$

for

$$\text{Cr}^{3+}:\text{ZnWO}_4, \quad g_x = g_y = 1.96, \quad g_z = 1.97, \quad D = 25.47 \text{ GHz} \quad E = -2.42 \text{ GHz}.$$

Each ion has an effective spin of 3/2.

The axes referred to in the spin-Hamiltonians are:

$\text{Mn}^{4+}:\text{TiO}_2$

$$x = [\bar{1}\bar{1}0]$$

$$y = [1\bar{1}0]$$

$$x = [001] \text{ (c-axis)}$$

$\text{Cr}^{3+}:\text{ZnWO}_4$

y axis along monoclinic b axis,
x and z in (010) plane with z
rotated 4.2° from +a towards
+c axis.

When the usual 4x4 Pauli spin matrices are substituted into (1) we obtain

$$(2) \begin{bmatrix} 3/2 A_1 \cos \theta + D & \frac{\sqrt{3}}{2} A_2 \sin \theta e^{-i\varphi} \sqrt{3} E & 0 \\ \frac{\sqrt{3}}{2} A_2 \sin \theta e^{i\varphi} & 1/2 A_1 \cos \theta - D & A_2 \sin \theta e^{-i\varphi} \sqrt{3} E \\ \sqrt{3} E & A_2 \sin \theta e^{i\varphi} & -1/2 A_1 \cos \theta - D & \frac{\sqrt{3}}{2} A_2 \sin \theta e^{-i\varphi} \\ 0 & \sqrt{3} E & \frac{\sqrt{3}}{2} A_2 \sin \theta e^{i\varphi} & -3/2 A_1 \cos \theta + D \end{bmatrix}$$

where $A_1 = g_z \beta H$, $A_2 = g_x \beta H = g_y \beta H$. H is the magnitude of the d.c. magnetic field, and θ and φ are the usual spherical coordinate angles; θ is measured from the z axis and φ is measure of the projection of \vec{H} in the $x-y$ plane from the x axis (Fig. 1.)

Energy Levels and Transition Probabilities

The energy levels (measured in frequency units) are simply the eigenvalues of (2), determined as a function of H , θ and φ . The transition probabilities are obtained from the eigenvectors $|1\rangle, |2\rangle, |3\rangle, |4\rangle$, (where the indices stand for increasing values of energy) through

$$|\langle i | s_x | j \rangle|^2, |\langle i | s_y | j \rangle|^2, |\langle i | s_z | j \rangle|^2,$$

where

$$s_x = \frac{1}{2} \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix}, \quad s_y = \frac{1}{2} \begin{bmatrix} 0 & -i\sqrt{3} & 0 & 0 \\ i\sqrt{3} & 0 & -i2 & 0 \\ 0 & i2 & 0 & -i\sqrt{3} \\ 0 & 0 & i\sqrt{3} & 0 \end{bmatrix},$$

(3)

$$s_z = \frac{1}{2} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

are the 4×4 Pauli spin matrices.

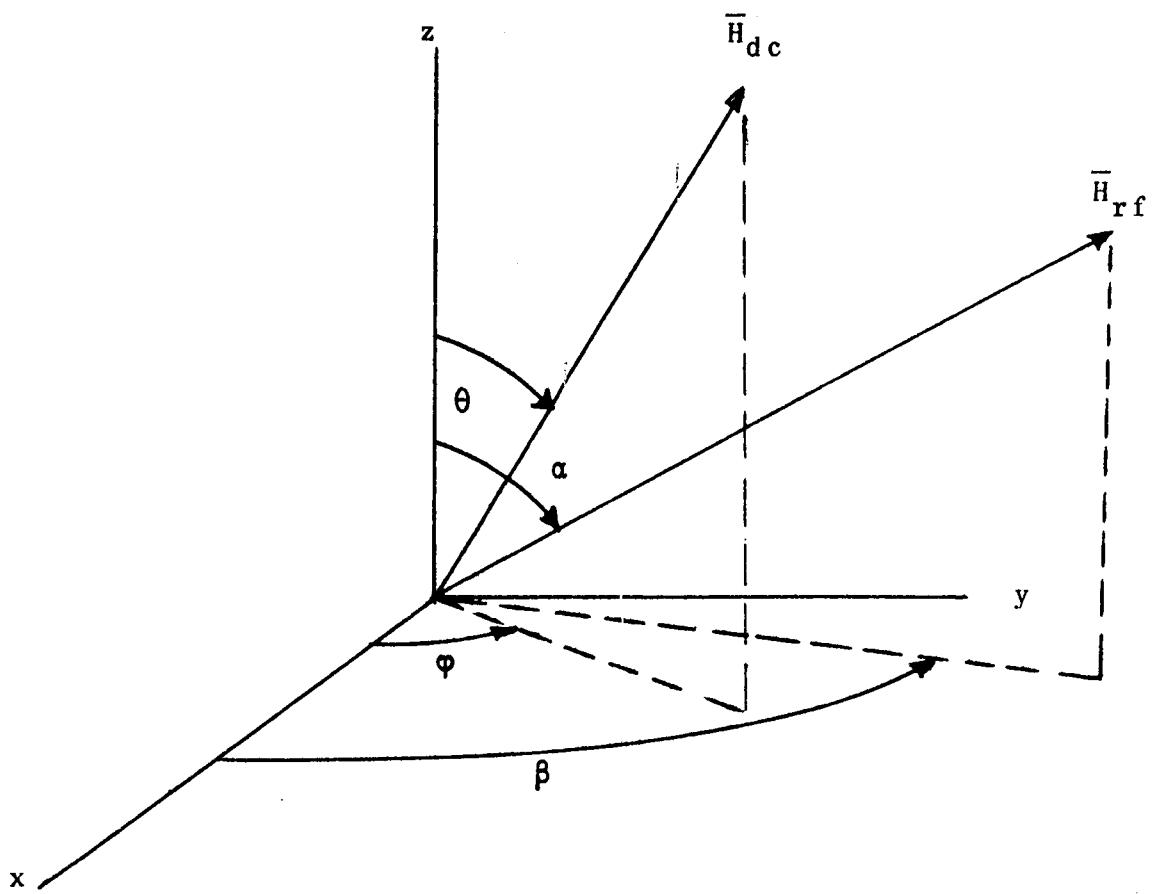


Fig. 1

ORIENTATION OF \bar{H}_{dc} AND \bar{H}_{rf}

If the eigenvector $|j\rangle$ is given by

$$(4) \quad |j\rangle = \begin{bmatrix} a_j \\ b_j \\ c_j \\ d_j \end{bmatrix},$$

then

$$(5) \quad \begin{aligned} \langle i | s_x | j \rangle &= \sqrt{3}/2 (a_i^* b_j + b_i^* a_j + c_i^* d_j + d_i^* c_j) + b_i^* c_j + c_i^* b_j \\ \langle i | s_y | j \rangle &= i\sqrt{3}/2 (-a_i^* b_j + b_i^* a_j - c_i^* d_j + d_i^* c_j) + i(-b_i^* c_j + c_i^* b_j) \\ \langle i | s_z | j \rangle &= 3/2 (a_i^* a_j - d_i^* d_j) + \frac{1}{2} (b_i^* b_j - c_i^* c_j), \end{aligned}$$

where * denotes complex conjugate.

For a linearly polarized r.f. magnetic field oriented at spherical coordinate angles (α, β) with respect to the coordinate axes as in Fig. 1, the transition probability between $|i\rangle$ and $|j\rangle$ is proportional to

$$(6) \quad \begin{aligned} &g_{\perp}^2 \sin^2 \alpha \cos^2 \beta |s_{x_{ij}}|^2 + g_{\perp}^2 \sin^2 \alpha \sin^2 \beta |s_{y_{ij}}|^2 + g_z^2 \cos^2 \alpha |s_{z_{ij}}|^2 \\ &+ g_{\perp}^2 \sin^2 \alpha \cos \beta \sin \beta (s_{x_{ij}}^* s_{y_{ij}} + s_{y_{ij}}^* s_{x_{ij}}) \\ &+ g_{\perp} g_z \sin \alpha \cos \alpha \cos \beta (s_{z_{ij}} s_{x_{ij}}^* + s_{z_{ij}}^* s_{x_{ij}}) + g_{\perp} g_z \sin \alpha \cos \alpha \\ &\sin \beta (s_{y_{ij}} s_{z_{ij}}^* + s_{y_{ij}}^* s_{z_{ij}}) \end{aligned}$$

where $g_{\perp} = g_x = g_y$.

Computation

In order that the computer be able to work with complex numbers, all entries in (2) of the form $a + jb$ were written as 2×2 matrices

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

which means that the computer is really working with an 8×8 matrix.

The computation of eigenvectors (and eigenvalues) was based on an iteration scheme described in Crandall (1956), Sections 2.7 and 2.8.

The expression (6) for the transition probability for linearly polarized radiation at arbitrary orientation (α, β) was maximized with respect to α and β , with H_{dc} , θ and φ treated as parameters. The maximum values of (6) are plotted and the values of (α, β) yielding the maxima are tabulated.

Tables and Graphs

Table 1 presents those values of α and β which maximize (6) for $Mn^{4+}:TiO_2$. The orientation, (θ, φ), and strength, H , of the external d.c. magnetic field are shown as parameters. H is in kilogauss. The symbols 1-2, 1-3, etc. stand for the 1-2 transition, 1-3 transition, etc. All energy levels are labeled in ascending order in the low-field limit. Table 2 is a similar presentation for $Cr^{3+}:ZnWO_4$.

Figs. 2-10 are the energy levels of $Mn^{4+}:TiO_2$ (measured in GHz) versus H (in kilogauss) with θ and φ treated as parameters. The corresponding curves for $Cr^{3+}:ZnWO_4$ are shown in Figs. 11-19.

For x-, y-, or z-polarized radiation the corresponding transition probabilities between the various energy levels are shown in Figs. 20-46 for $Mn^{4+}:TiO_2$ and Figs. 47-73 for $Cr^{3+}:ZnWO_4$. The maximum transition probabilities for arbitrarily oriented, linearly polarized radiation (eqn. 6), are shown in Figs. 74-82 for $Mn^{4+}:TiO_2$ and Figs. 83-91 for $Cr^{3+}:ZnWO_4$. It should be remembered when using the curves for the various transition probabilities that they are labeled according to transitions between energy level curves, each energy level curve being numbered in ascending order of energy in the weak (d.c.) field limit. This point is important because if two energy level curves cross for a certain value of H , the orders of their energies becomes reversed in the weak and high field limits, but they still carry the weak field label.

Conclusions

We have plotted the most important features for maser operation that can be obtained from the spin-Hamiltonians for $Mn^{4+}:TiO_2$ and $Cr^{3+}:ZnWO_4$. The energy levels determine the operating frequencies, both signal and pump, while the transition probabilities (or, to be more precise, the transition matrix elements) determine the gain at the signal frequency and also the ease with which saturation of energy levels at the pump frequency occurs. Thus, from a study of the curves contained in this report one may determine the operating points for masers employing $Mn^{4+}:TiO_2$ and $Cr^{3+}:ZnWO_4$.

It should be pointed out, in addition, that other important parameters not obtainable from a spin-Hamiltonian and therefore not included herein are the various relaxation times T_1 , T_2 (variously called "spin-spin", "spin-lattice", "longitudinal and transverse", etc.). They determine line-widths and pump transition characteristics, as well.

Acknowledgment

We appreciate the assistance of Profs. J. W. Barnstead and A. R. Schmidt in the computation of the eigenvalues and eigenvectors. Thanks are also due to Dr. D. E. Criss and Mr. Roy Rose for assistance with the IBM 1130 computer of the Waters Computing Center, Rose Polytechnic Institute.

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2. Crandall, S.H., "Engineering Analysis", McGraw-Hill Book Co., New York, 1956.
3. Kurtz, S.K. and Nilsen, W.G., "Paramagnetic Resonance Spectra of Cr^{3+} in ZnO_4 ", Phys. Rev. 128:1586-1588, November, 1962.

Table 1

THETA=0												PHI = ANYTHING																							
1-2						1-3						1-4						2-3						2-4						3-4					
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA									
1	106	180	86	180	120	180	0	0	108	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
2	88	0	89	180	138	180	0	0	88	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
3	90	180	90	180	29	0	180	0	97	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
4	90	180	90	180	17	0	180	0	90	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
5	90	180	90	180	7	0	180	0	96	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
6	90	180	90	180	179	0	180	0	79	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
7	90	180	90	0	8	180	180	0	74	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
8	90	180	90	180	166	0	180	0	108	180	29	0	29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
9	90	180	90	0	18	180	180	0	70	0	155	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
10	90	180	58	180	180	0	51	0	90	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90							
THETA=45												PHI = 0																							
1-2						1-3						1-4						2-3						2-4						3-4					
H	ALPHA	BETA	ALPHA	BETA	ALPHA	ALPHA	BETA	ALPHA	ALPHA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA							
1	45	180	80	180	36	0	29	0	83	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
2	47	180	80	180	37	0	157	180	85	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
3	49	180	80	180	37	0	16	0	87	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
4	50	180	98	0	36	0	8	0	89	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
5	128	0	96	0	35	0	179	0	89	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
6	127	0	86	180	146	180	9	180	88	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
7	127	0	88	180	146	180	16	180	87	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
8	127	0	90	0	148	180	159	0	86	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
9	127	0	90	0	146	180	26	180	86	0	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
10	127	0	86	0	26	0	29	180	96	180	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90					
THETA=45												PHI = 30																							
1-2						1-3						1-4						2-3						2-4						3-4					
H	ALPHA	BETA	ALPHA	BETA	ALPHA	ALPHA	BETA	ALPHA	ALPHA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA	ALPHA	ALPHA	BETA	BETA							
1	133	10	82	176	136	150	144	154	84	175	99	110	99	110	99	110	99	110	99	110	99	110	99	110	99	110	99	110	99	110	99	110			
2	132	9	82	177	135	148	150	157	86	175	98	110	98	110	98	110	98	110	98	110	98	110	98	110	98	110	98	110	98	110					
3	130	7	98	-2	134	146	158	161	88	175	97	113	97	113	97	113	97	113	97	113	97	113	97	113	97	113	97	113	97	113					
4	128	5	96	-1	133	145	168	166	89	175	95	115	95	115	95	115	95	115	95	115	95	115	95	115	95	115	95	115	95	115					
5	126	4	86	180	133	143	176	140	91	175	93	118	93	118	93	118	93	118	93	118	93	118	93	118	93	118	93	118	93	118					
6	55	183	89	180	133	141	9	175	91	176	93	121	93	121	93	121	93	121	93	121	93	121	93	121	93	121	93	121	93	121					
7	56	183	89	0	133	140	17	178	92	176	93	123	93	123	93	123	93	123	93	123	93	123	93	123	93	123	93	123	93	123					
8	56	183	87	0	134	137	156	-1	92	176	94	124	94	124	94	124	94	124	94	124	94	124	94	124	94	124	94	124	94	124					
9	56	184	85	1	134	136	28	183	93	176	95	124	95	124	95	124	95	124	95	124	95	124	95	124	95	124	95	124	95	124					
10	56	184	96	178	134	135	148	4	93	176	95	123	95	123	95	123	95	123	95	123	95	123	95	123	95	123	95	123	95	123					

Table 1. (continued)

H	THETA=45				PHI=60							
	1-2		1-3		1-4		2-3		2-4		3-4	
	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	128	22	86	177	125	150	128	155	87	176	109	129
2	127	21	85	178	124	145	135	150	88	176	108	130
3	127	17	86	179	125	143	141	154	89	177	105	132
4	125	12	88	180	124	144	152	160	90	178	101	135
5	121	7	89	0	124	142	169	168	90	1	96	140
6	117	4	84	-1	123	143	172	2	91	4	94	146
7	66	184	80	-3	124	141	30	152	93	5	93	151
8	68	184	12	174	124	140	37	170	95	7	94	153
9	112	4	103	172	124	140	135	-9	95	7	95	154
10	112	5	104	171	125	138	49	173	96	8	95	154

H	THETA=45				PHI=90							
	1-2		1-3		1-4		2-3		2-4		3-4	
	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	129	128	79	-4	120	143	16	46	89	180	91	173
2	135	100	79	-5	124	141	18	35	93	0	89	172
3	135	97	78	-5	126	131	20	25	90	180	92	172
4	90	180	78	-5	127	129	90	141	90	180	92	172
5	138	90	78	-5	133	105	27	10	90	0	92	172
6	90	0	79	-4	134	95	101	141	124	91	92	172
7	79	-4	90	180	93	177	33	7	92	172	90	180
8	79	-4	90	0	133	74	35	3	92	172	90	180
9	80	-3	89	-1	96	174	37	1	91	173	90	180
10	81	-3	89	-1	104	165	39	-1	91	173	90	180

H	THETA=90				PHI = 0							
	1-2		1-3		1-4		2-3		2-4		3-4	
	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	180	0	90	180	90	90	90	90	90	180	90	90
2	180	0	90	180	90	90	90	90	90	0	90	90
3	180	0	90	0	90	90	90	90	90	0	90	90
4	180	90	90	180	90	90	90	90	90	0	90	90
5	180	0	90	0	90	90	90	90	90	180	90	90
6	180	0	90	180	90	90	90	90	90	180	90	90
7	180	0	90	180	90	90	90	90	90	180	90	90
8	180	0	90	180	90	90	90	90	90	180	90	90
9	180	0	90	180	90	90	90	90	90	180	90	90
10	180	0	90	180	90	90	90	90	90	180	90	90

Table 1. (continued)

THETA=90												PHI=30													
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA		ALPHA	BETA										
1	180	0	90	173	90	58	90	64	90	170	90	120		180	0	90	175	90	56	90	69	90	170	90	122
2	180	0	90	175	90	56	90	69	90	170	90	122		180	0	90	177	90	55	90	75	90	170	90	123
3	180	0	90	179	90	55	90	82	90	171	90	125		0	0	90	0	90	55	90	88	90	172	90	126
4	0	0	90	179	90	55	90	82	90	171	90	125		180	0	90	1	90	55	90	93	90	174	90	126
5	0	0	90	0	90	55	90	88	90	172	90	126		180	0	90	2	90	55	90	98	90	175	90	127
6	180	0	90	1	90	55	90	93	90	174	90	126		0	0	90	3	90	55	90	12	90	176	90	127
7	180	0	90	2	90	55	90	98	90	175	90	127		0	0	90	3	90	56	90	105	90	177	90	127
8	0	0	90	3	90	55	90	12	90	176	90	127		180	0	90	3	90	56	90	107	90	178	90	127
9	0	0	90	3	90	56	90	105	90	177	90	127		180	0	90	4	90	56	90	107	90	178	90	127
10	180	0	90	4	90	56	90	107	90	178	90	127													
	THETA=90												PHI=60												
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA		ALPHA	BETA										
1	180	0	90	175	90	66	90	69	90	173	90	150		180	0	90	177	90	65	90	74	90	173	90	151
2	180	0	90	178	90	66	90	80	90	174	90	153		180	0	90	0	90	68	90	89	90	176	90	154
3	180	0	90	178	90	66	90	80	90	174	90	153		180	0	90	3	90	71	90	98	90	179	90	155
4	180	0	90	0	90	68	90	89	90	176	90	154		180	0	90	7	90	79	90	109	90	3	90	156
5	180	0	90	3	90	71	90	98	90	179	90	155		180	0	90	7	90	79	90	118	90	007	90	156
6	180	0	90	7	90	79	90	109	90	179	90	156		0	0	90	10	90	83	90	126	90	11	90	156
7	180	0	90	7	90	79	90	118	90	007	90	156		180	0	90	12	90	86	90	131	90	14	90	156
8	0	0	90	10	90	83	90	126	90	179	90	156		0	0	90	14	90	89	90	135	90	17	90	156
9	180	0	90	12	90	86	90	131	90	179	90	156		0	0	90	14	90	89	90	135	90	17	90	156
10	0	0	90	14	90	89	90	135	90	179	90	156													
	THETA=90												PHI=90												
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4	
	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA		ALPHA	BETA										
1	0	0	90	21	90	144	90	90	90	180	90	180		180	0	90	4	90	116	90	90	90	3	90	172
2	180	0	90	4	90	116	90	90	90	180	90	180		180	0	90	14	90	131	90	90	90	180	90	180
3	180	0	90	14	90	124	90	90	90	180	90	180		180	0	90	12	90	117	90	90	90	180	90	180
4	180	0	90	12	90	124	90	90	90	180	90	180		180	0	90	9	90	121	90	90	90	0	90	180
5	180	0	90	9	90	121	90	90	90	180	90	180		180	0	90	6	90	117	90	90	90	180	90	180
6	180	0	90	6	90	117	90	90	90	180	90	180		180	0	90	4	90	113	90	90	90	180	90	180
7	180	0	90	4	90	113	90	90	90	180	90	180		180	0	90	2	90	111	90	90	90	0	90	180
8	0	0	90	1	90	108	90	90	90	180	90	180		180	0	90	0	90	106	90	90	90	180	90	180
9	180	0	90	1	90	108	90	90	90	180	90	180		180	0	90	0	90	108	90	90	90	180	90	180
10	180	0	90	0	90	106	90	90	90	180	90	180													

Table 2

THETA=0												PHI = ANYTHING														
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4		
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	H	ALPHA	BETA											
1	81	0	90	90	78	0	0	0	90	90	167	180	1	90	90	90	90	90	90	90	90	90	90	90	90	
2	88	0	90	90	178	180	0	0	90	90	31	0	2	90	90	90	90	90	90	90	90	90	90	90	90	
3	90	0	90	90	158	180	0	0	90	90	58	0	3	90	90	90	90	90	90	90	90	90	90	90	90	
4	90	0	90	90	9	0	0	0	90	90	90	90	4	90	90	90	90	90	90	90	90	90	90	90	90	
5	90	0	90	90	176	180	0	0	90	90	90	90	5	90	90	90	90	90	90	90	90	90	90	90	90	
6	90	0	90	90	179	150	0	0	90	90	90	90	6	90	90	90	90	90	90	90	90	90	90	90	90	
7	86	0	90	90	90	90	0	0	90	90	90	90	7	90	90	90	90	90	90	90	90	90	90	90	90	
8	90	0	90	90	90	90	0	0	90	90	90	90	8	90	90	90	90	90	90	90	90	90	90	90	90	
9	95	0	90	90	90	90	0	0	90	90	90	90	9	90	90	90	90	90	90	90	90	90	90	90	90	
10	77	180	90	90	90	90	0	0	90	90	90	90	10	90	90	90	90	90	90	90	90	90	90	90	90	
	THETA=45												PHI = 0													
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4		
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	H	ALPHA	BETA											
1	90	90	90	90	90	90	90	90	90	90	51	180	1	90	90	90	90	90	90	90	90	90	90	90	90	
2	90	90	90	90	90	90	78	180	90	90	137	0	2	90	90	90	90	90	90	90	90	90	90	90	90	
3	90	90	90	90	90	90	81	180	90	90	146	0	3	90	90	90	90	90	90	90	90	90	90	90	90	
4	90	90	90	180	90	90	90	180	90	90	49	0	4	90	90	90	90	90	90	90	90	90	90	90	90	
5	90	90	90	90	90	90	90	180	90	90	165	0	5	90	90	90	90	90	90	90	90	90	90	90	90	
6	90	90	90	90	90	90	86	0	90	90	179	-1	6	90	90	90	90	90	90	90	90	90	90	90	90	
7	90	90	90	90	90	90	78	0	90	90	137	180	7	90	90	90	90	90	90	90	90	90	90	90	90	
8	90	90	67	0	90	90	68	0	90	90	90	90	8	90	90	90	90	90	90	90	90	90	90	90	90	
9	90	90	110	180	90	90	126	180	90	90	90	90	9	90	90	90	90	90	90	90	90	90	90	90	90	
10	90	90	76	0	90	90	38	0	90	90	90	90	10	90	90	90	90	90	90	90	90	90	90	90	90	
	THETA=45												PHI=30													
	1-2		1-3		1-4		2-3		2-4		3-4			1-2		1-3		1-4		2-3		2-4		3-4		
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	H	ALPHA	BETA											
1	85	126	92	91	81	100	77	112	93	91	135	42	1	85	126	92	91	93	91	135	42	1	85	126	92	91
2	85	125	90	88	84	94	79	180	94	91	140	46	2	85	125	90	88	94	91	140	46	2	85	125	90	88
3	85	125	89	88	83	99	76	154	95	90	138	50	3	85	125	89	88	95	90	138	50	3	85	125	89	88
4	85	125	89	87	83	101	74	100	95	89	136	56	4	85	125	89	87	95	89	136	56	4	85	125	89	87
5	85	125	87	84	84	100	71	94	96	88	134	65	5	85	125	87	84	96	88	134	65	5	85	125	87	84
6	86	125	85	79	85	98	66	88	97	87	130	74	6	86	125	85	79	97	87	130	74	6	86	125	85	79
7	86	124	83	73	86	97	60	80	97	86	124	80	7	86	124	83	73	97	86	124	80	7	86	124	83	73
8	87	124	82	66	87	95	51	72	98	84	117	82	8	87	124	82	66	98	84	117	82	8	87	124	82	66
9	88	123	82	60	87	96	40	66	98	83	113	82	9	88	123	82	60	98	83	113	82	9	88	123	82	60
10	88	123	84	55	88	94	28	62	99	81	110	80	10	88	123	84	55	99	81	110	80	10	88	123	84	55

Table 2. (continued)

	THETA=45				PHI=60							
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	86	156	93	91	74	109	71	101	96	92	135	68
2	87	163	92	86	73	105	68	100	97	92	134	73
3	86	155	89	85	78	101	66	99	98	90	136	77
4	86	154	89	88	80	99	61	96	99	90	130	77
5	87	154	87	86	81	99	56	94	100	89	128	80
6	87	153	86	84	82	99	50	91	101	88	178	82
7	87	153	86	82	85	87	43	88	103	89	116	95
8	88	152	86	79	83	98	35	86	102	86	118	83
9	88	152	87	77	84	97	26	85	102	85	118	83
10	88	152	88	72	84	98	18	92	102	85	117	83

	THETA=45				PHI=90							
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	90	1	94	89	74	92	69	90	97	90	135	90
2	90	0	92	89	76	84	65	89	98	91	133	92
3	90	0	90	90	77	89	61	90	99	90	131	91
4	90	0	89	90	79	89	56	90	100	90	129	90
5	90	0	88	90	80	90	51	90	101	90	127	90
6	90	0	87	90	81	90	44	90	102	90	124	90
7	90	0	87	90	81	90	37	90	103	90	123	90
8	90	0	87	90	82	90	30	90	103	90	121	90
9	90	0	88	90	83	90	22	90	103	90	120	90
10	89	180	88	85	85	95	14	90	103	90	119	90

	THETA=90				PHI = 0							
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	90	90	90	90	90	0	90	0	90	90	0	0
2	90	90	90	90	90	0	90	0	90	90	0	0
3	90	90	90	90	90	0	90	0	90	90	0	0
4	90	90	90	90	90	0	90	0	90	90	0	0
5	90	90	90	90	90	0	90	0	90	90	0	0
6	90	90	90	90	90	0	90	0	90	90	0	0
7	90	90	90	90	90	0	90	0	90	90	0	0
8	BAD POINT											
9	90	90	90	90	90	0	90	0	90	90	0	0
10	90	90	90	90	90	0	90	0	90	90	0	0

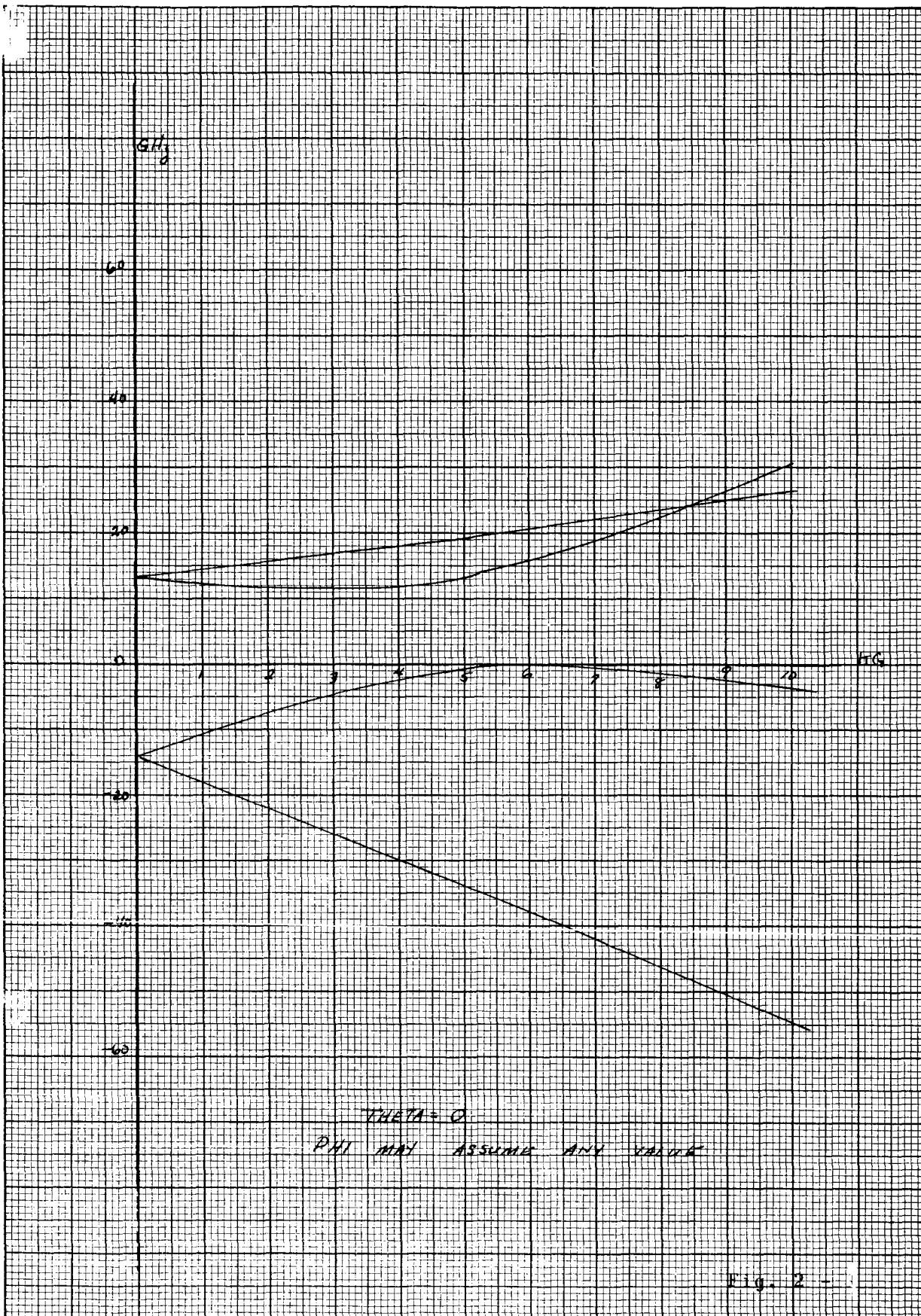
Table 2. (continued)

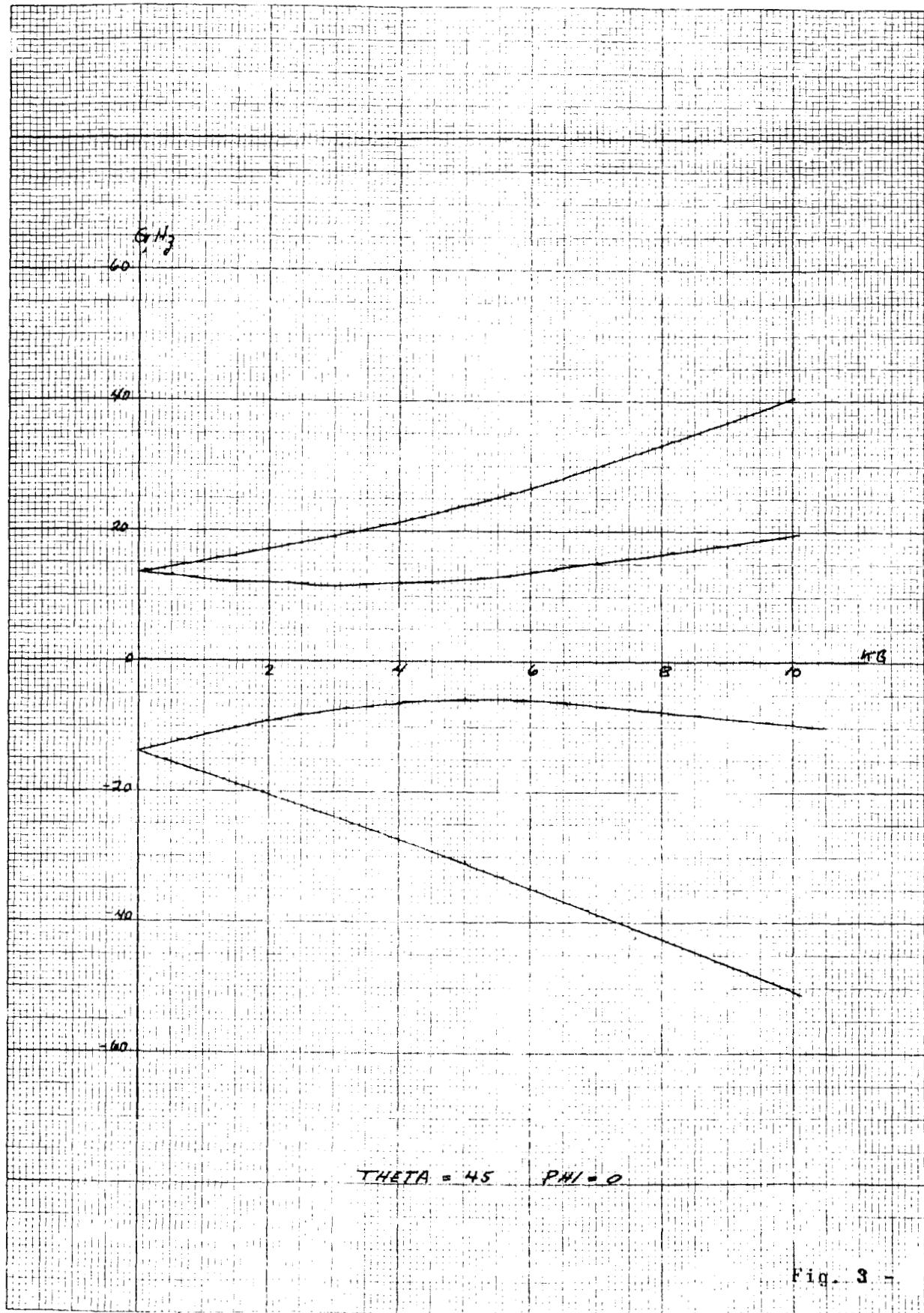
	THETA=90						PHI=30					
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	90	120	90	94	90	27	90	5	90	109	0	0
2	90	120	90	93	90	9	90	3	90	95	0	0
3	90	120	90	91	90	5	90	1	90	92	0	0
4	90	120	90	88	90	1	90	177	90	88	0	0
5	90	119	90	85	90	175	90	172	90	83	0	0
6	90	119	90	80	90	170	90	166	90	79	0	0
7	90	119	90	75	90	164	90	159	90	74	0	0
8	90	119	90	69	90	159	90	152	90	69	0	0
9	90	119	90	64	90	154	90	146	90	64	0	0
10	90	118	90	60	90	150	90	141	90	60	0	0

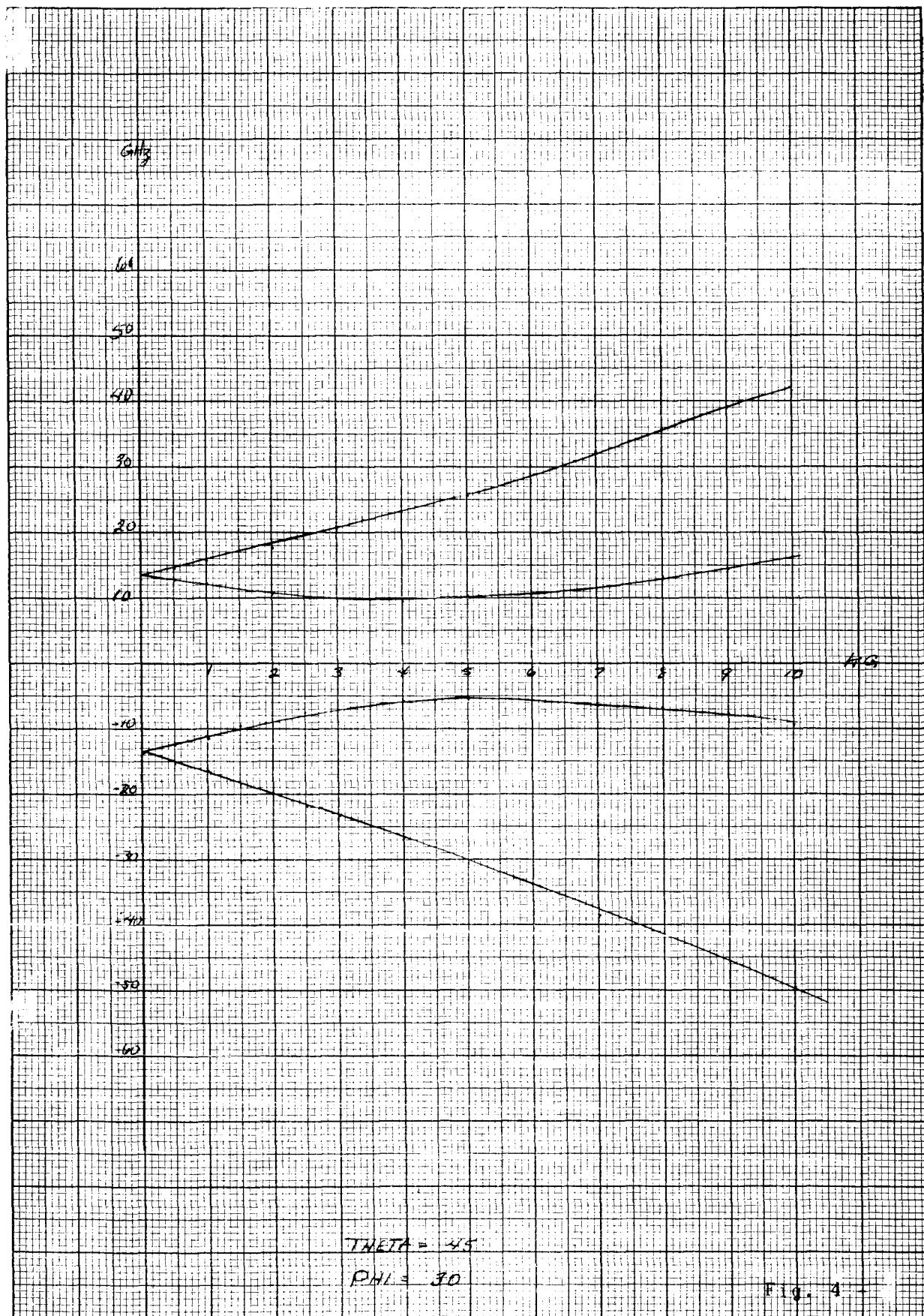
	THETA=90						PHI=60					
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	90	148	90	94	90	43	90	5	90	120	0	0
2	90	150	90	93	90	12	90	4	90	98	0	0
3	90	150	90	92	90	7	90	1	90	93	0	0
4	90	150	90	90	90	3	90	179	90	90	0	0
5	90	149	90	88	90	1	90	176	90	87	0	0
6	90	149	90	86	90	178	90	173	90	85	0	0
7	90	149	90	84	90	175	90	170	90	82	0	0
8	90	149	90	82	90	174	90	168	90	80	0	0
9	90	149	90	81	90	172	90	165	90	79	0	0
10	90	148	90	79	90	171	90	163	90	78	0	0

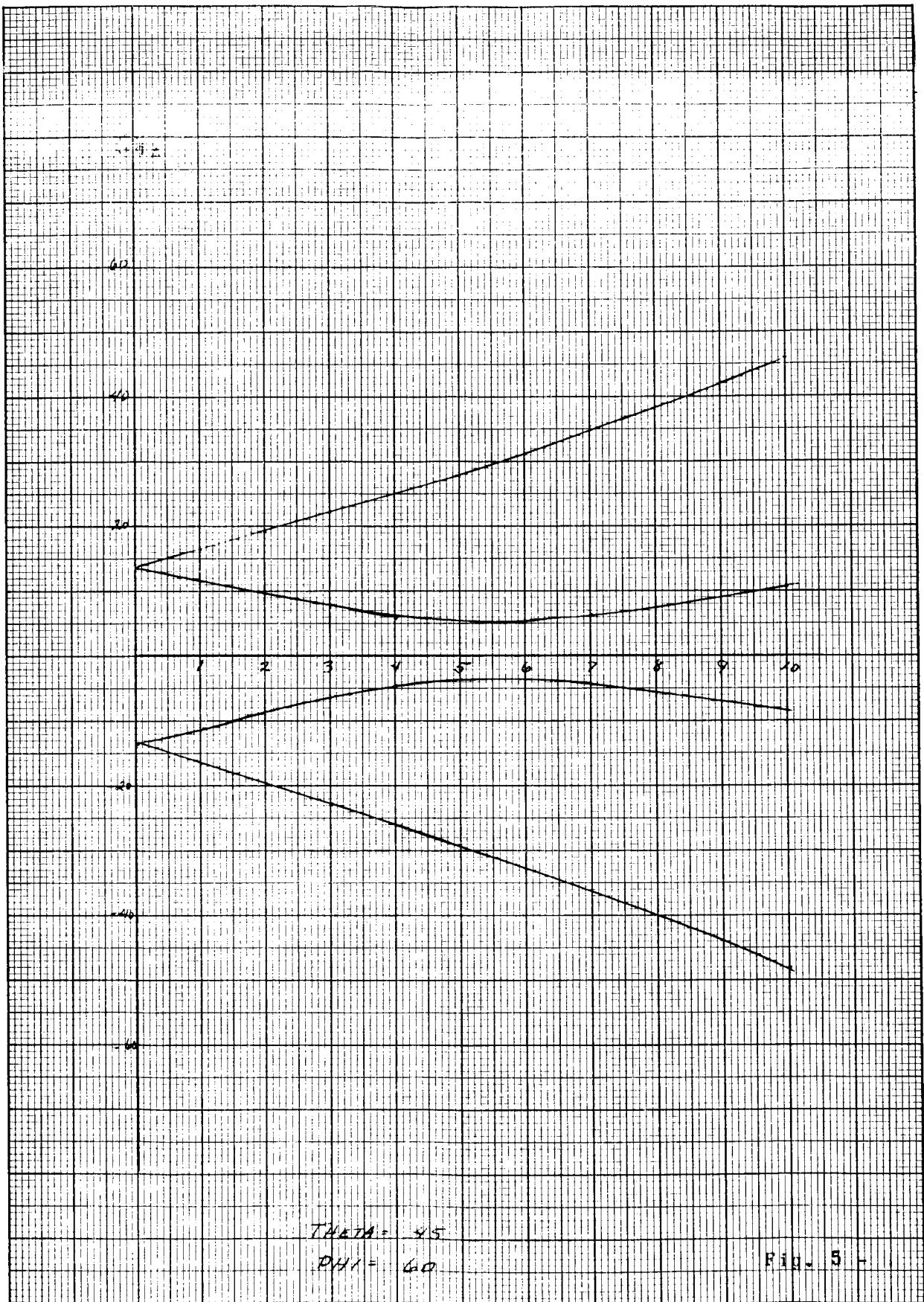
	THETA=90						PHI=90					
	1-2		1-3		1-4		2-3		2-4		3-4	
H	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA	ALPHA	BETA
1	90	0	90	52	90	0	90	131	90	90	0	0
2	90	5	90	43	90	167	90	133	90	90	0	0
3	90	1	90	53	90	176	90	135	90	90	0	0
4	90	0	90	58	90	0	90	137	90	90	0	0
5	90	0	90	61	90	0	90	140	90	90	0	0
6	90	0	90	63	90	0	90	142	90	90	0	0
7	90	0	90	66	90	0	90	146	90	90	0	0
8	90	0	90	67	90	1	90	149	90	90	0	0
9	90	0	90	71	90	179	90	152	90	90	0	0
10	90	0	90	73	90	-1	90	156	90	90	0	0

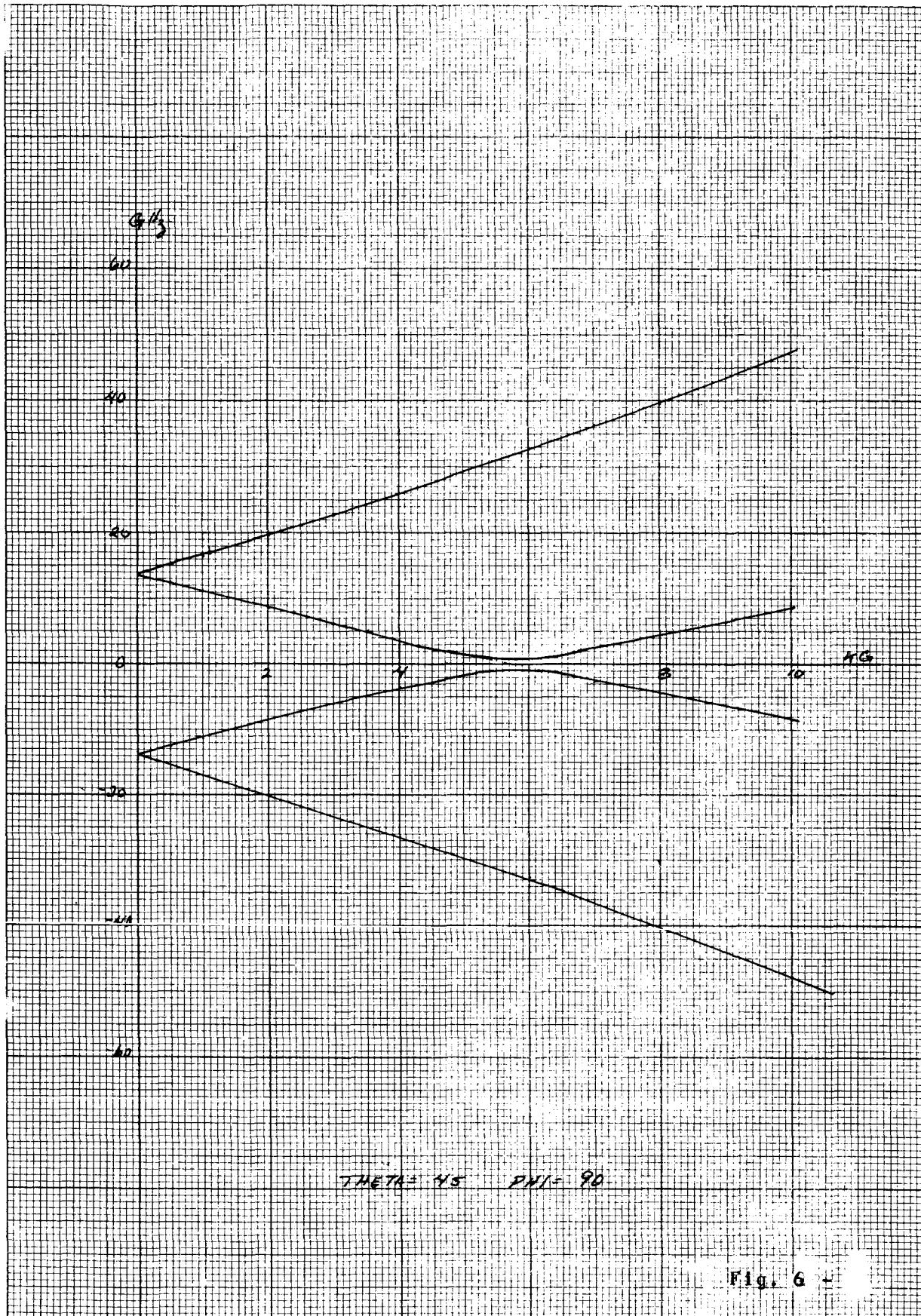
Figs. 2-10
ENERGY LEVELS FOR Mn^{4+} : TiO_2











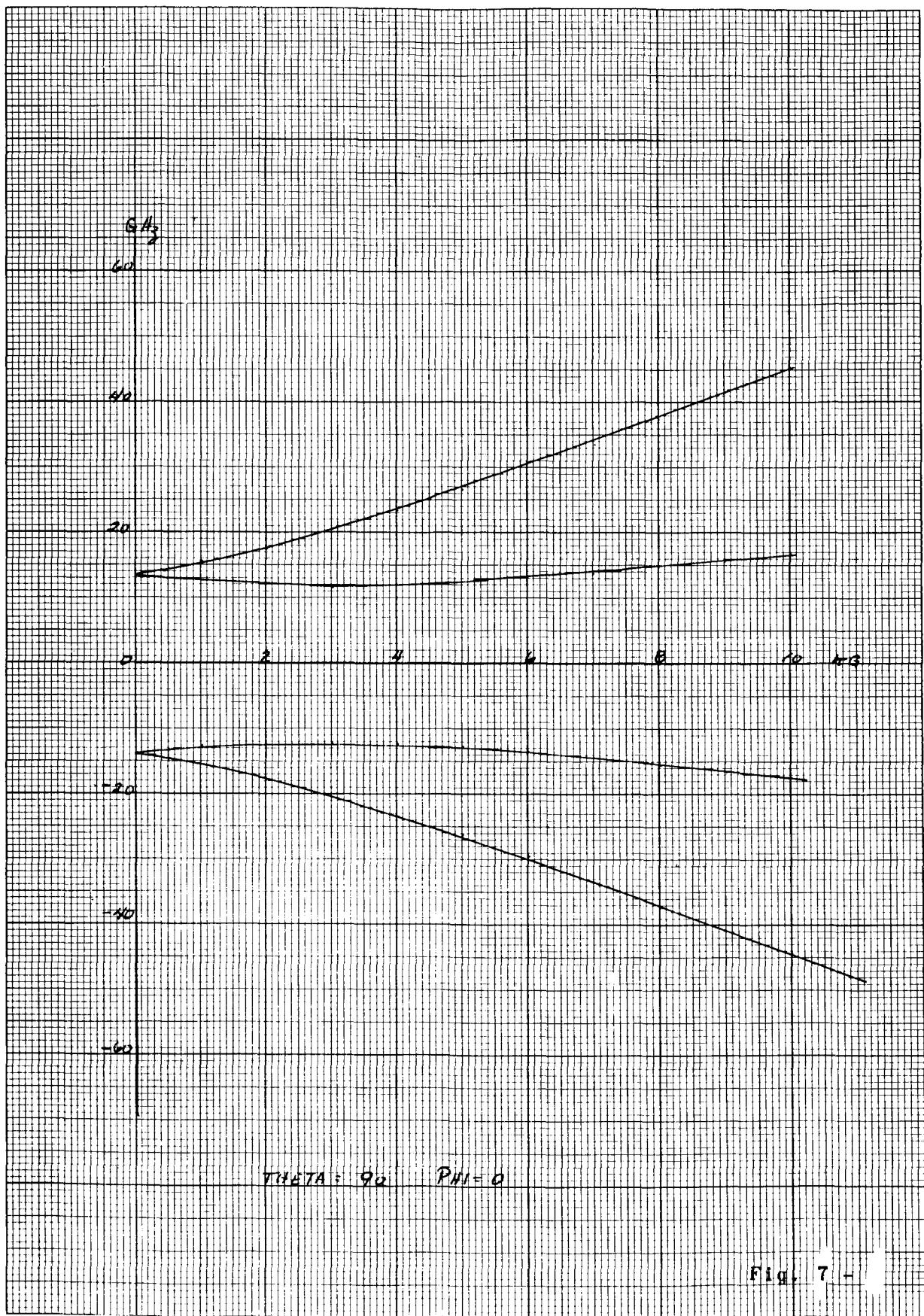
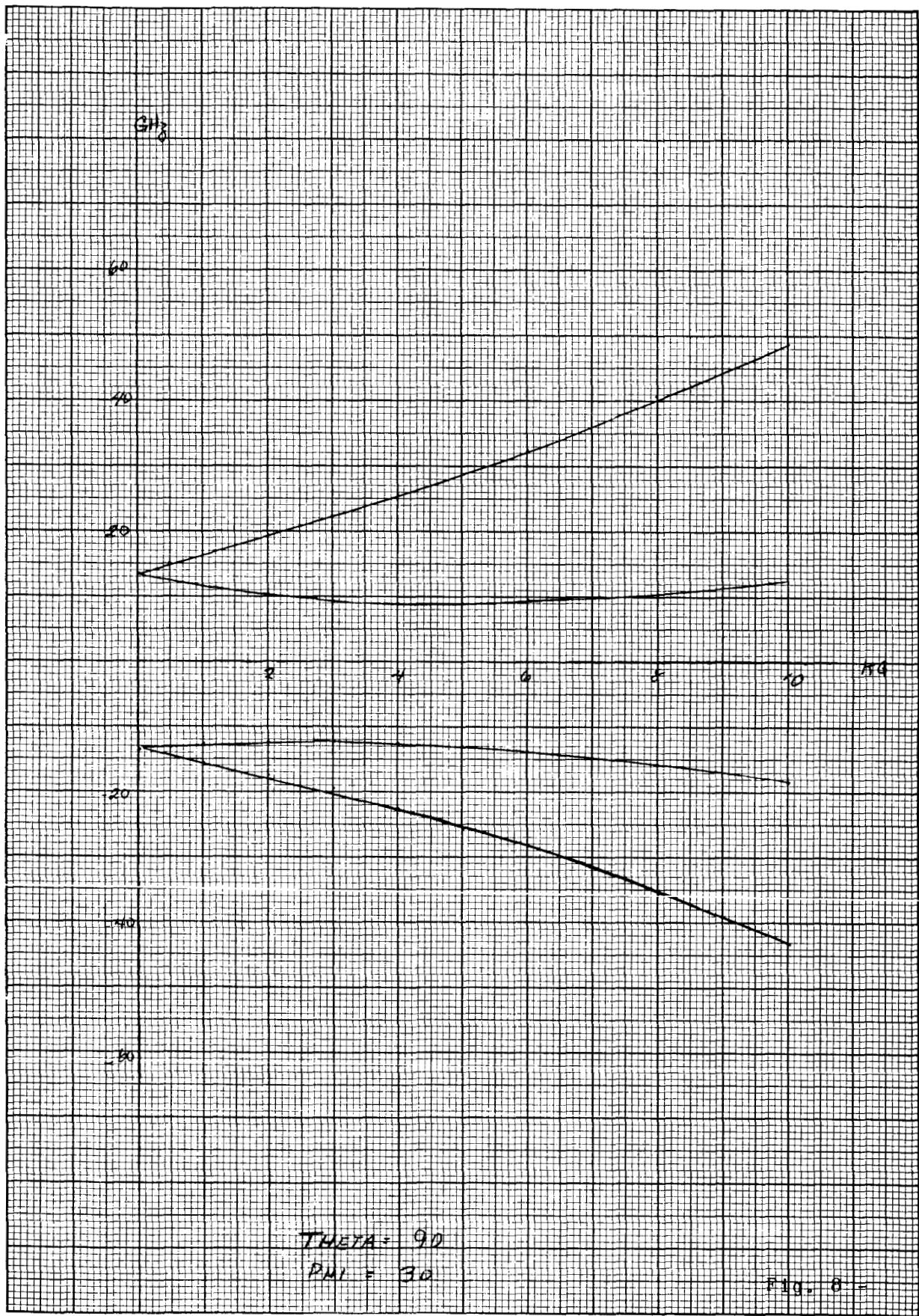
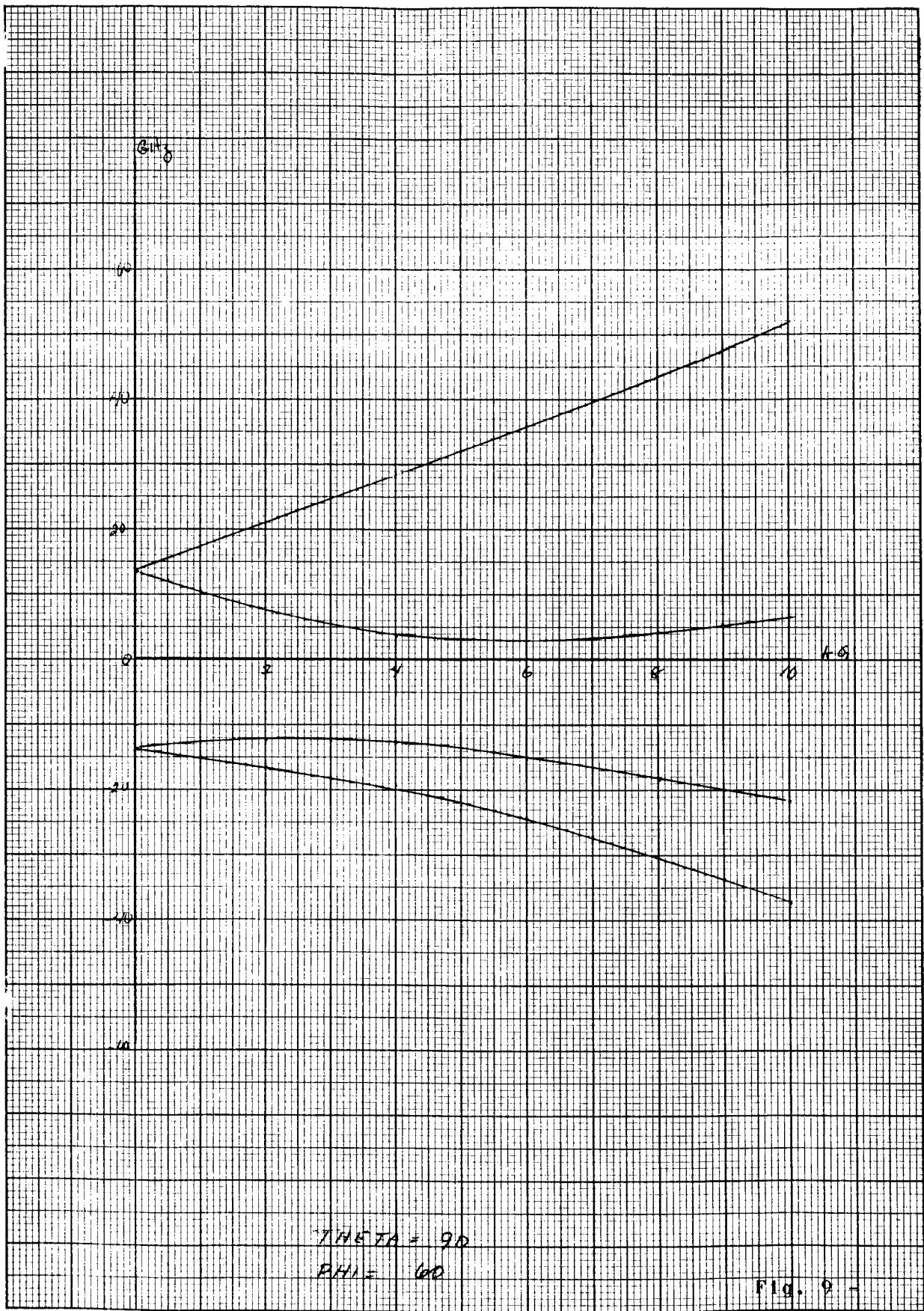
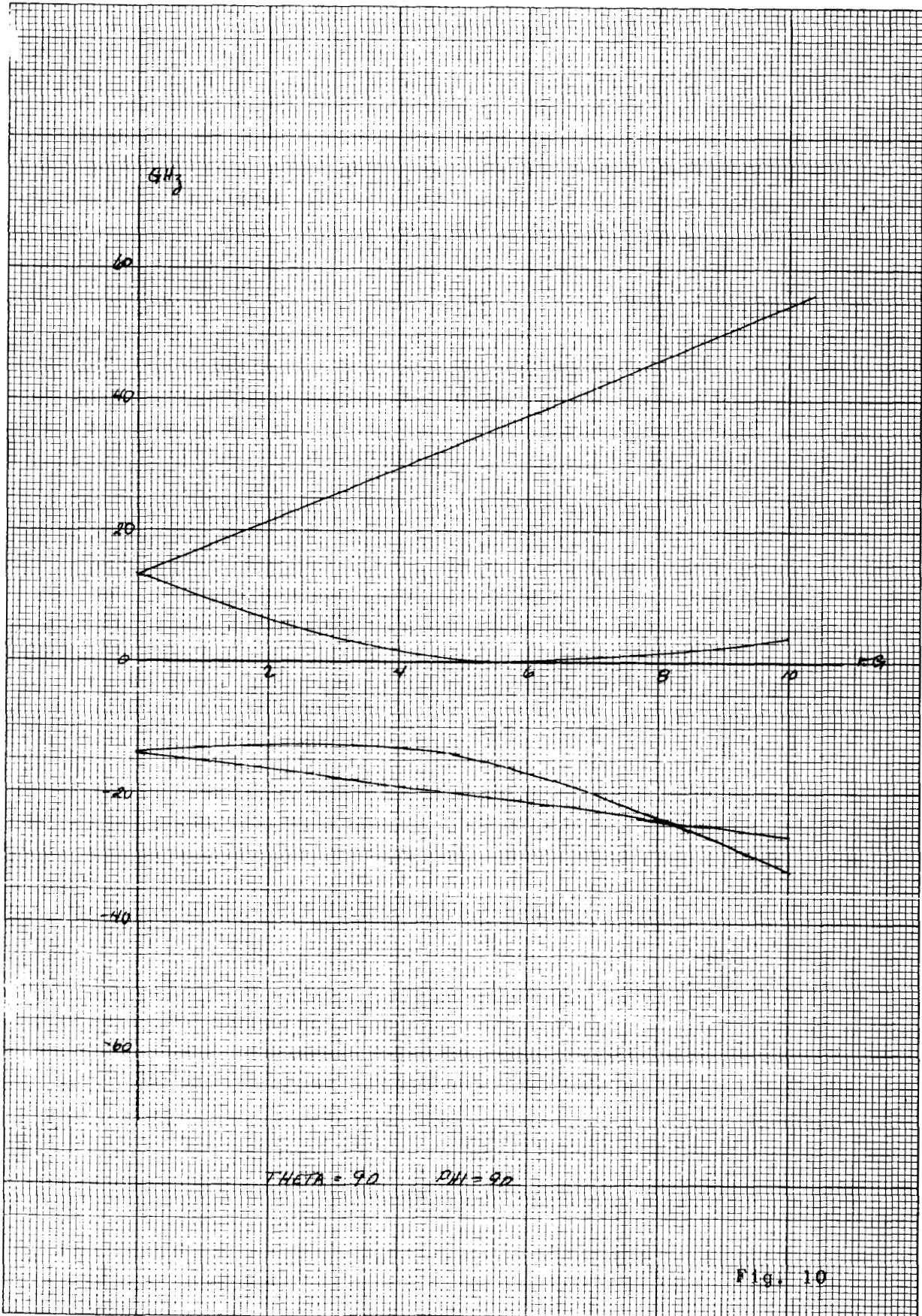


Fig. 7 -







Figs. 11-19

ENERGY LEVELS FOR Cr³⁺:ZnWO₄

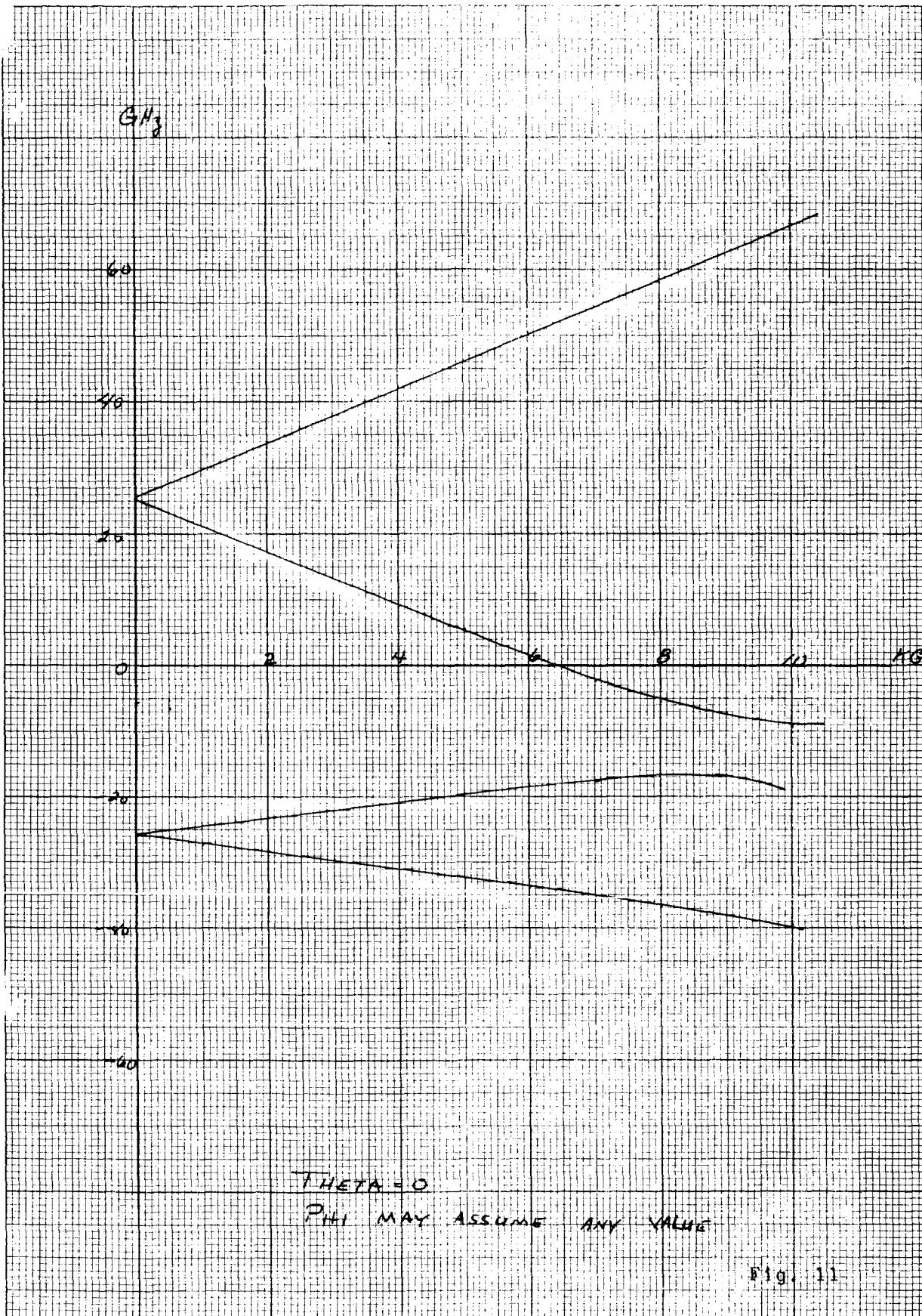
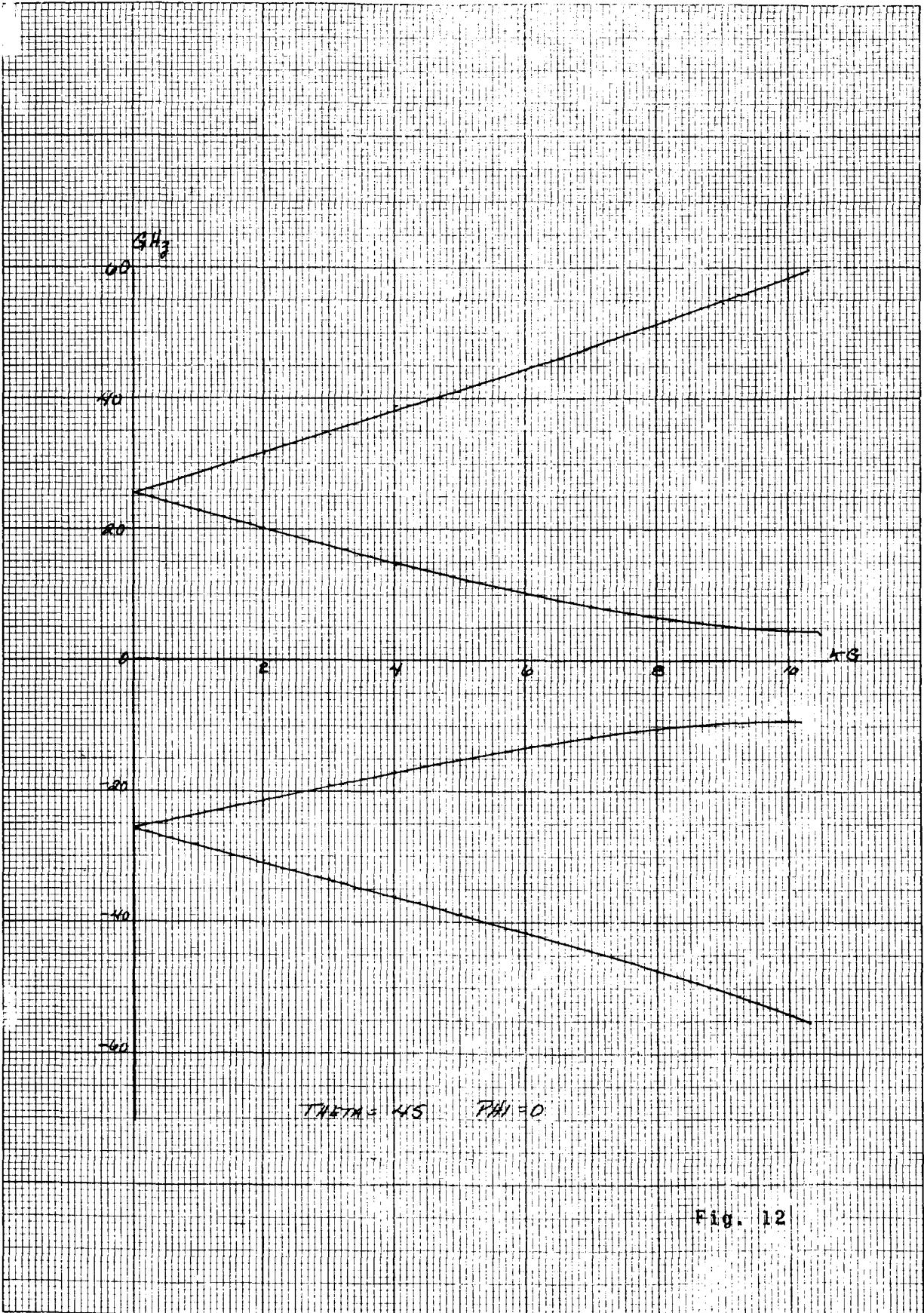
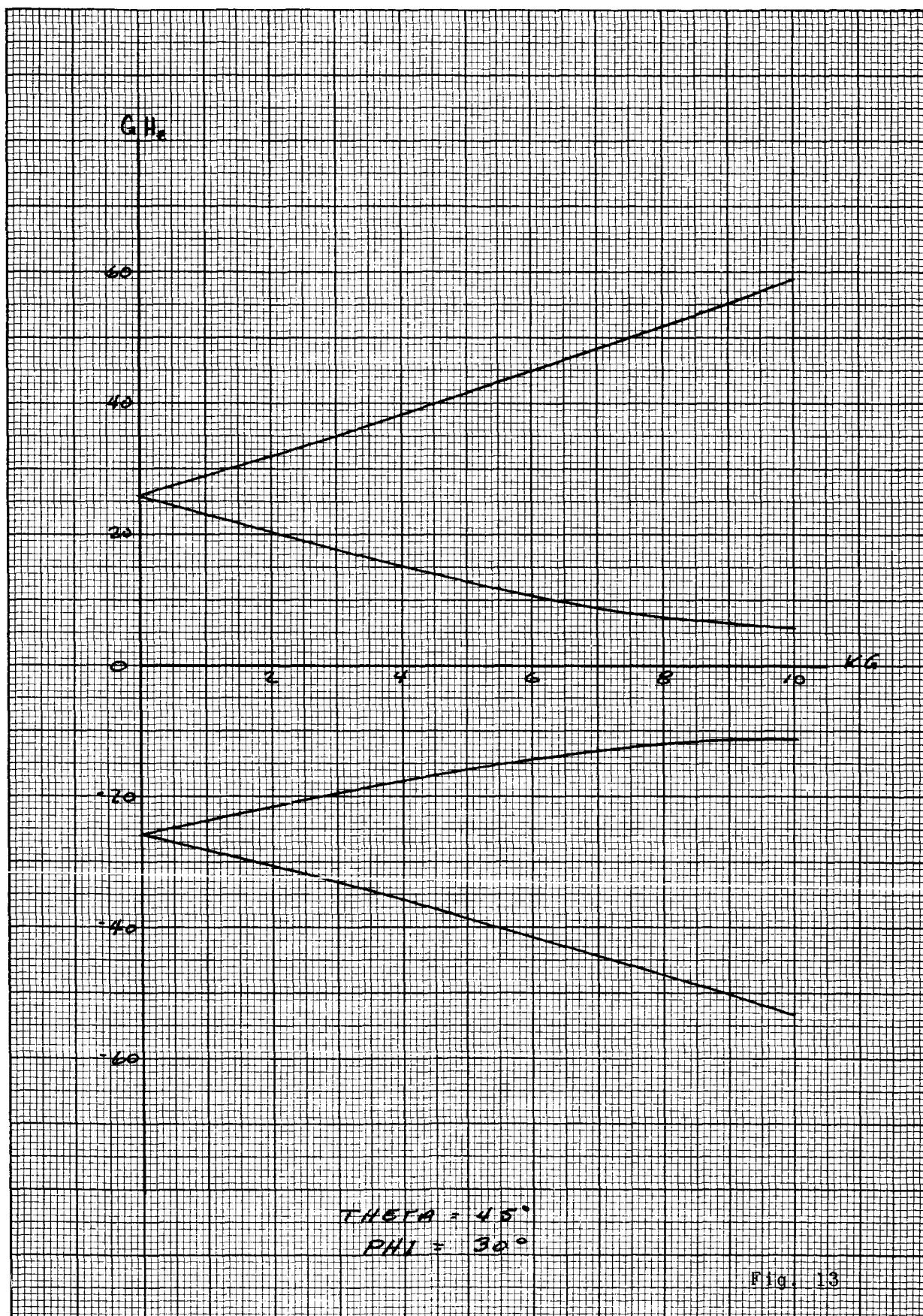
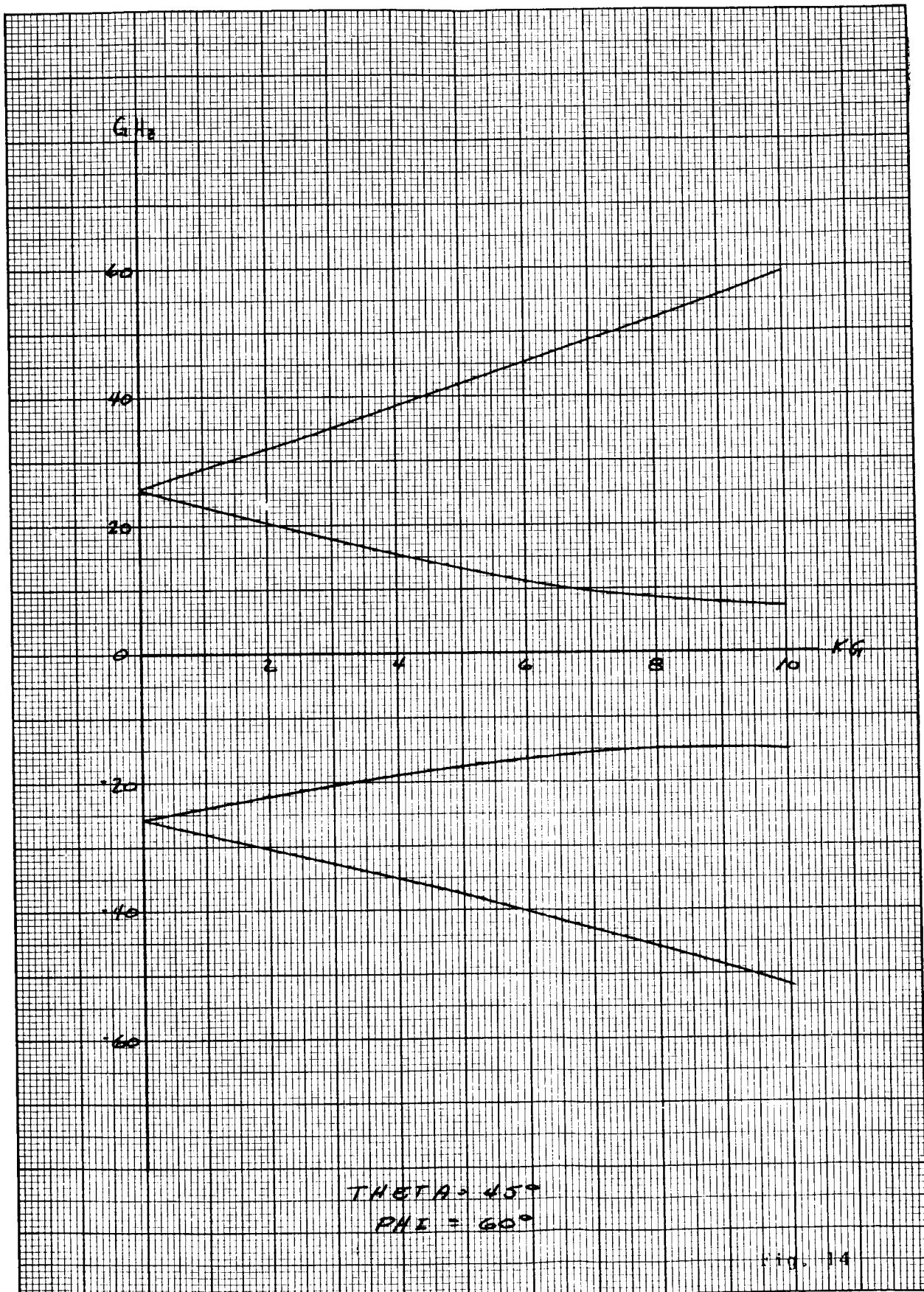


Fig. 11







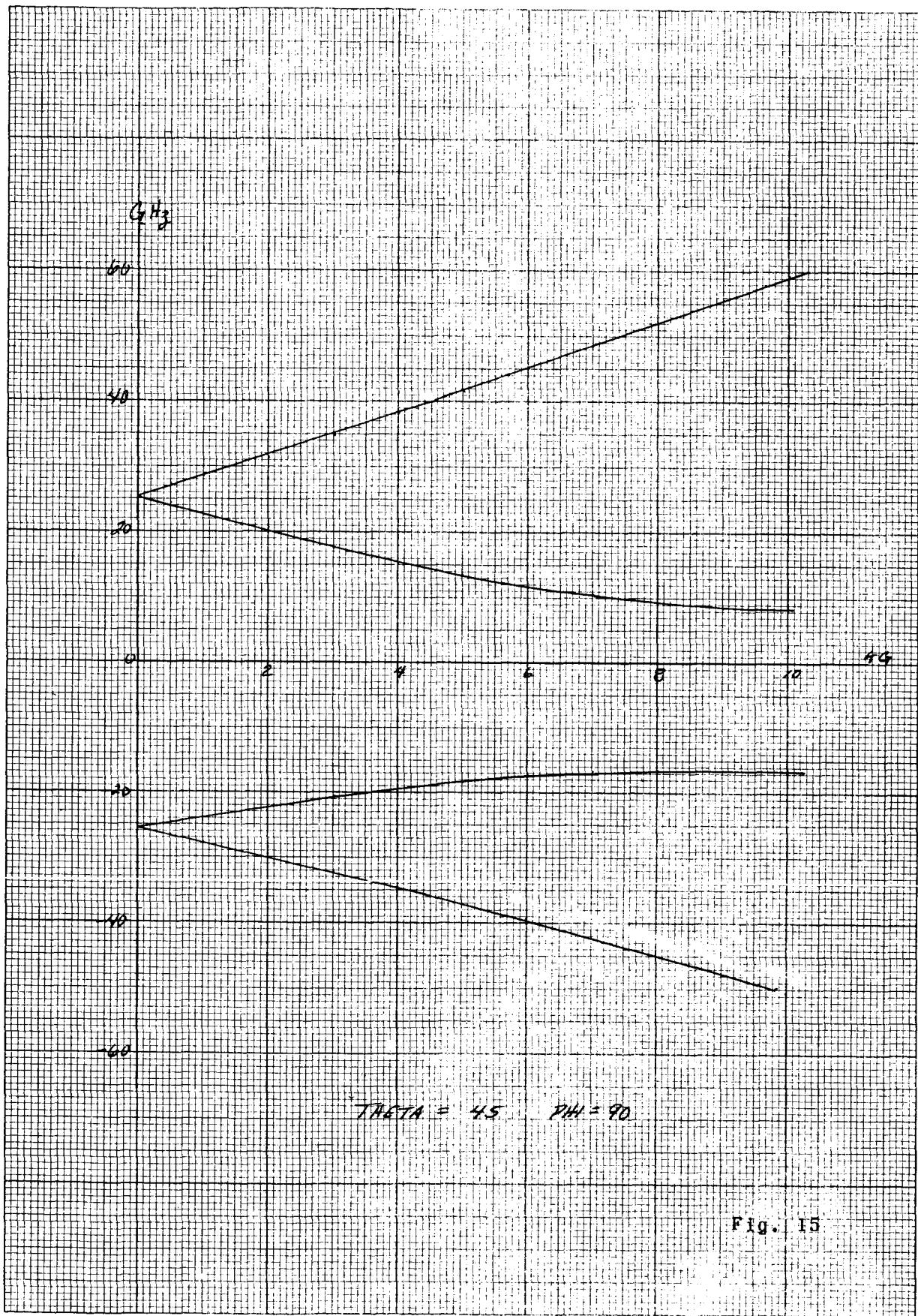


Fig. 15

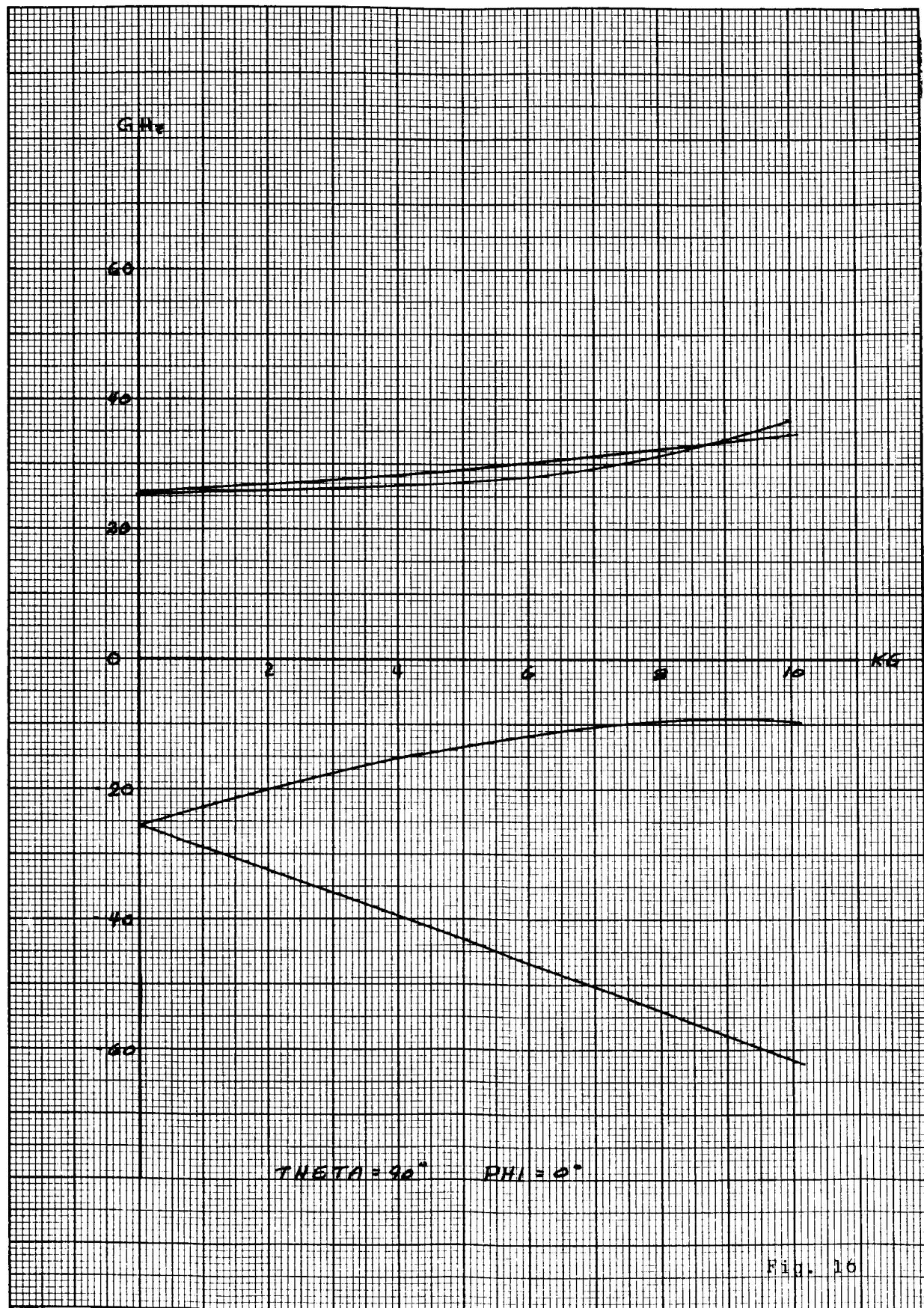
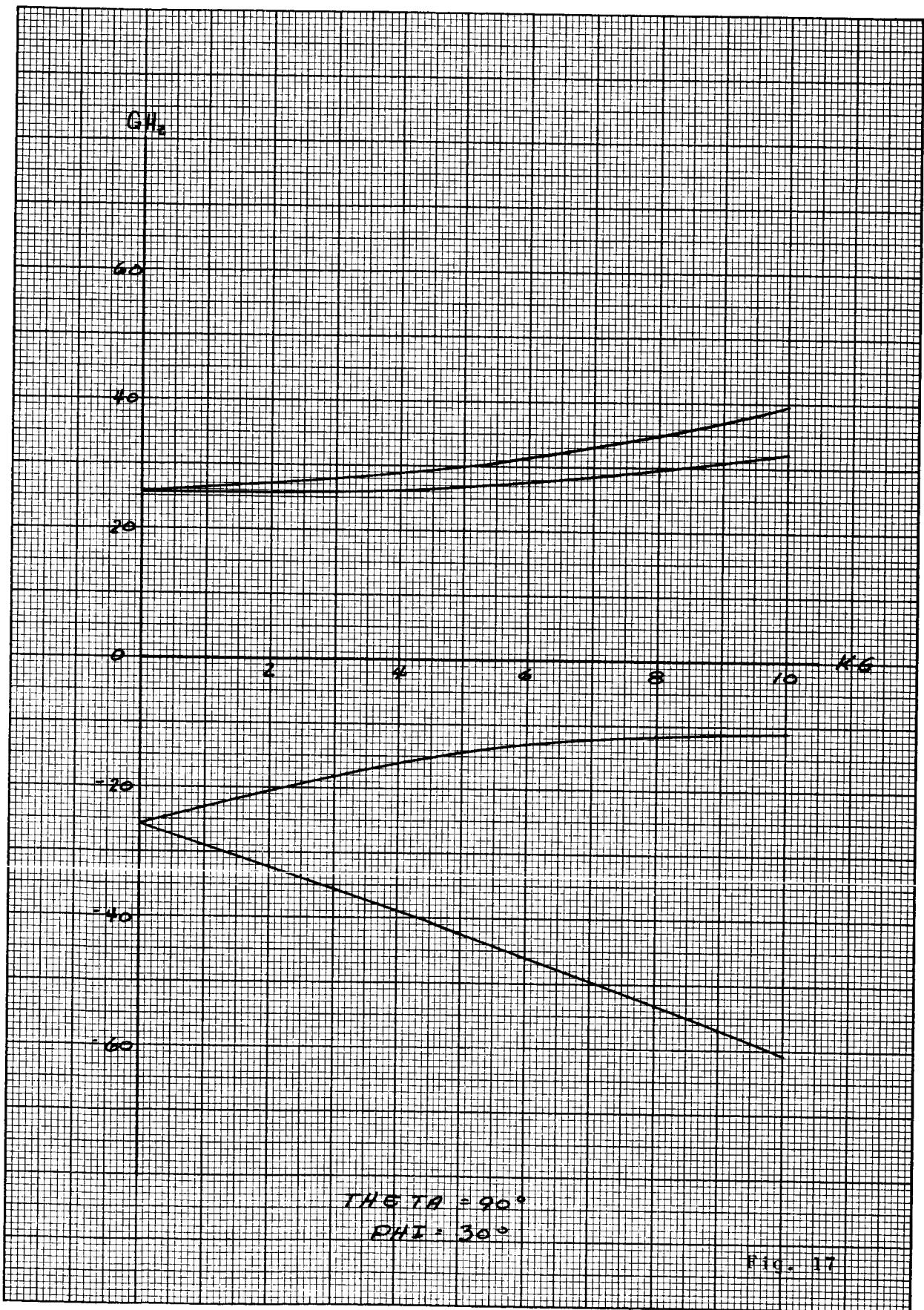
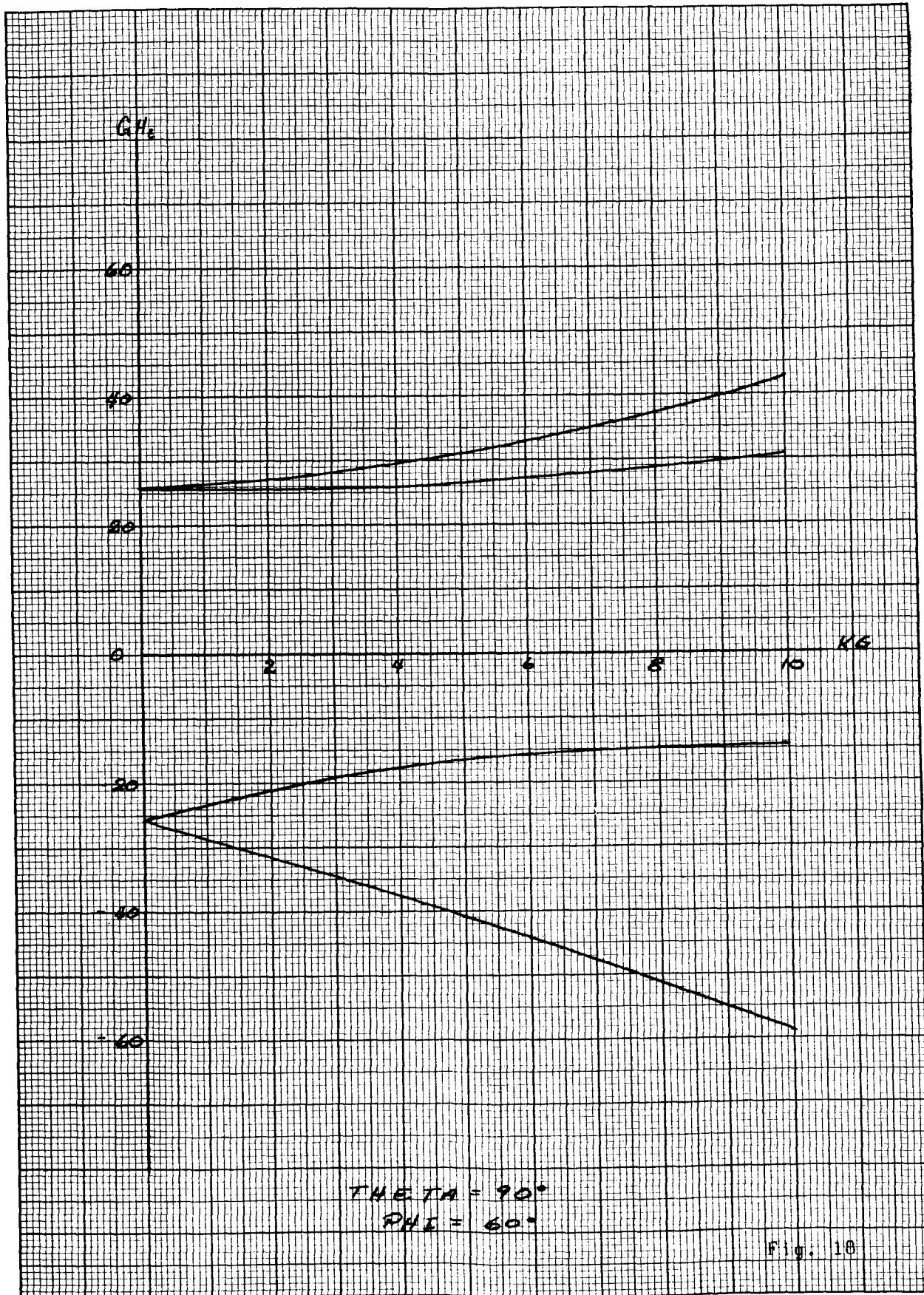
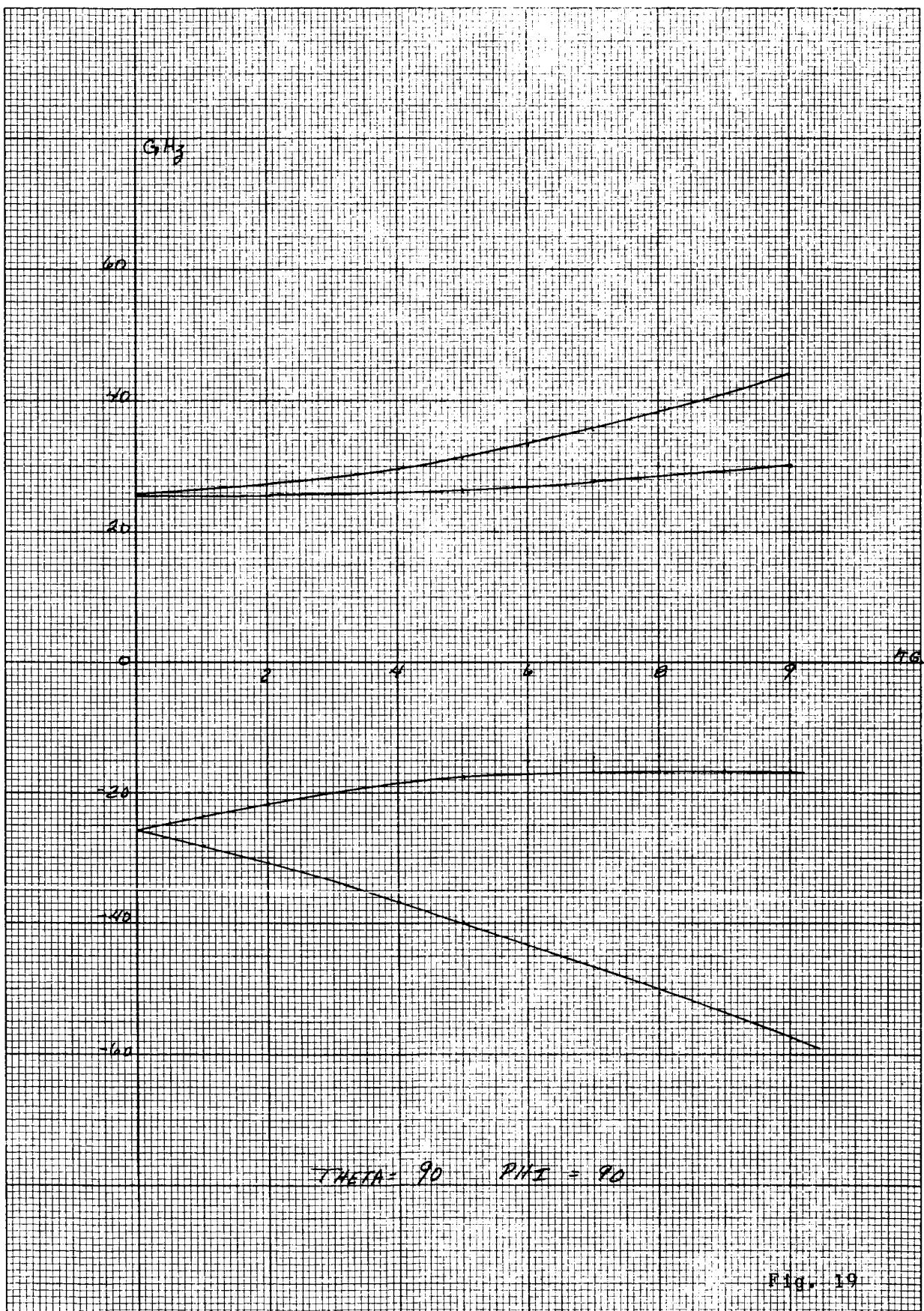


Fig. 16







Figs. 20-46

$|s_{x_{ij}}|^2$, $|s_{y_{ij}}|^2$ and $|s_{z_{ij}}|^2$ for $Mn^{4+}:TiO_2$

H.F. along X-axis
 H_{dc} $\theta=0$ ϕ : any angle

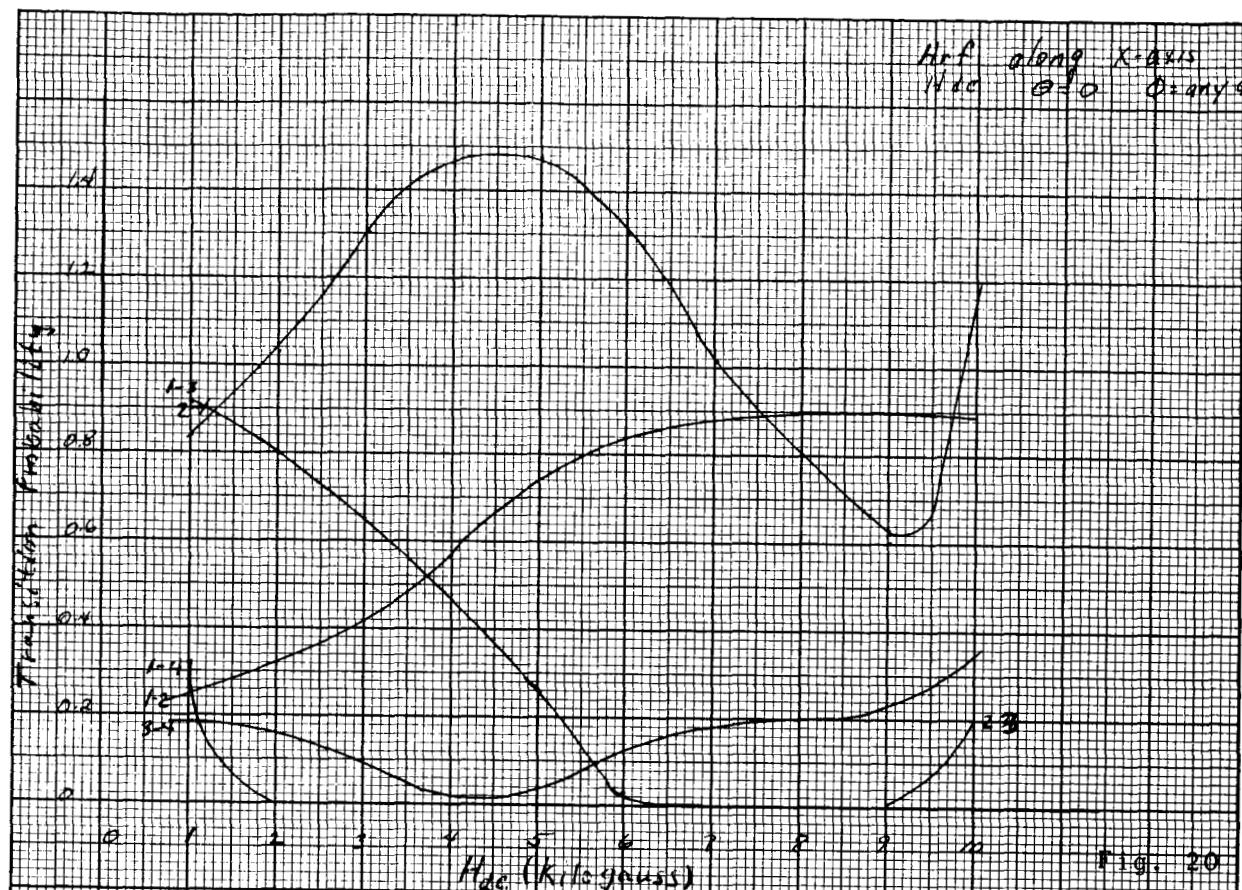


FIG. 20

H.F. along X-axis
 H_{dc} $\theta=45^\circ$ $\phi=0$

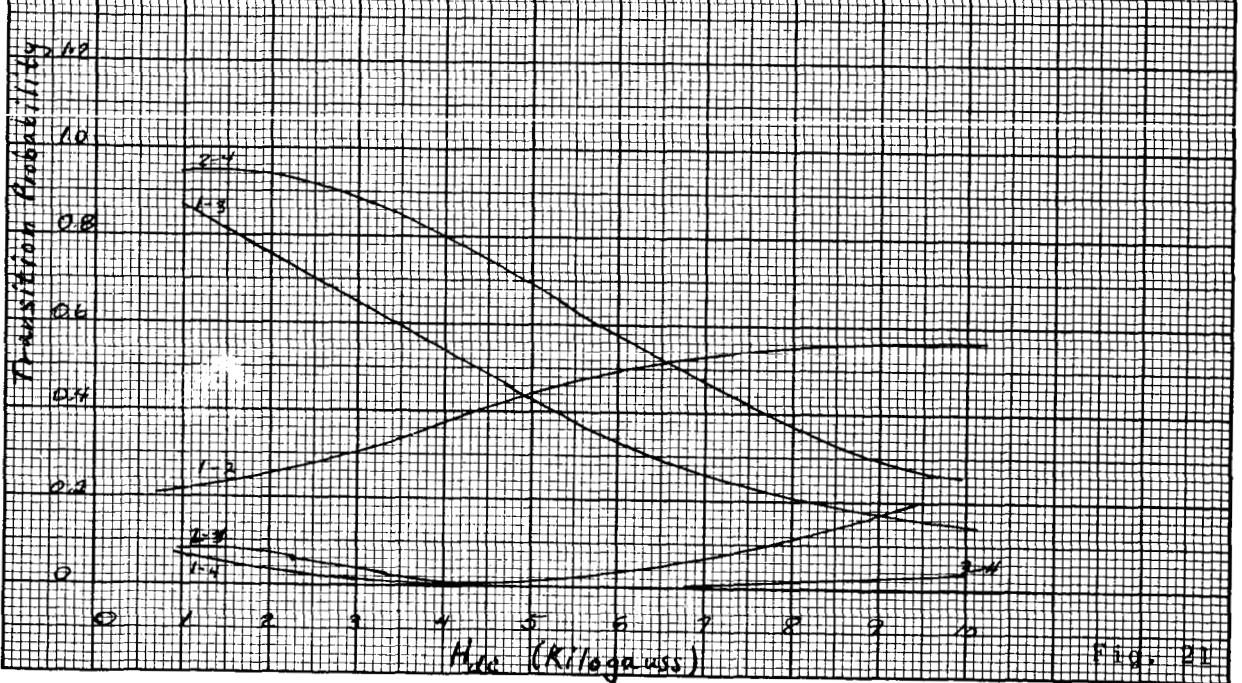
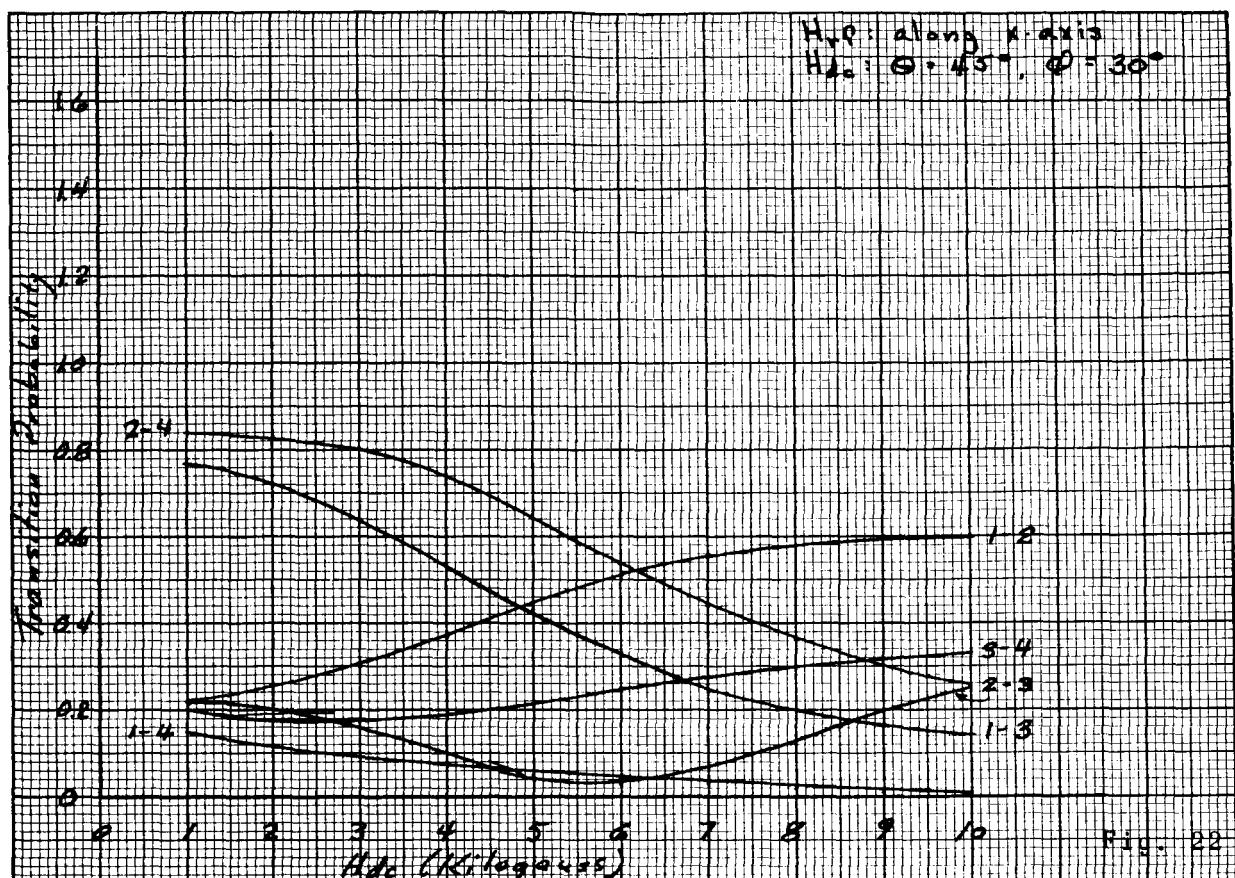
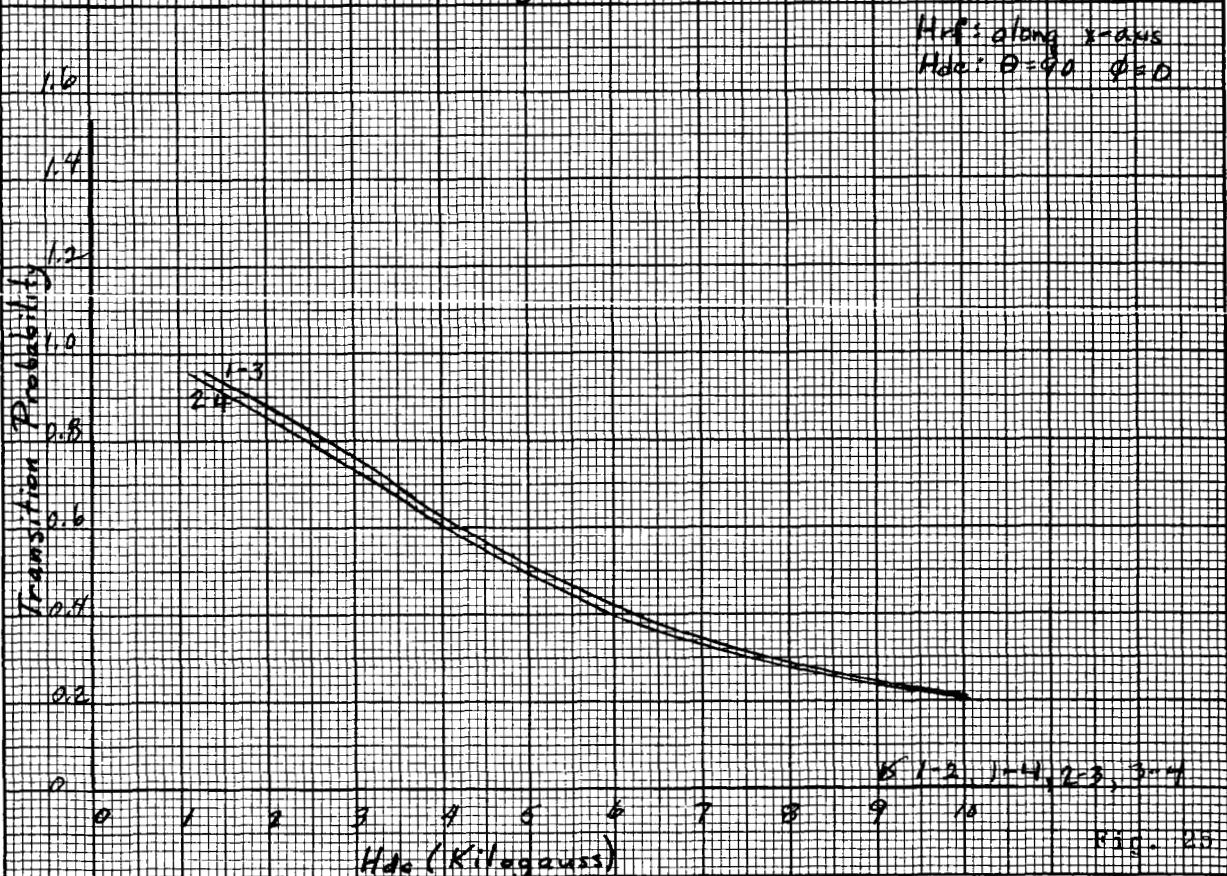
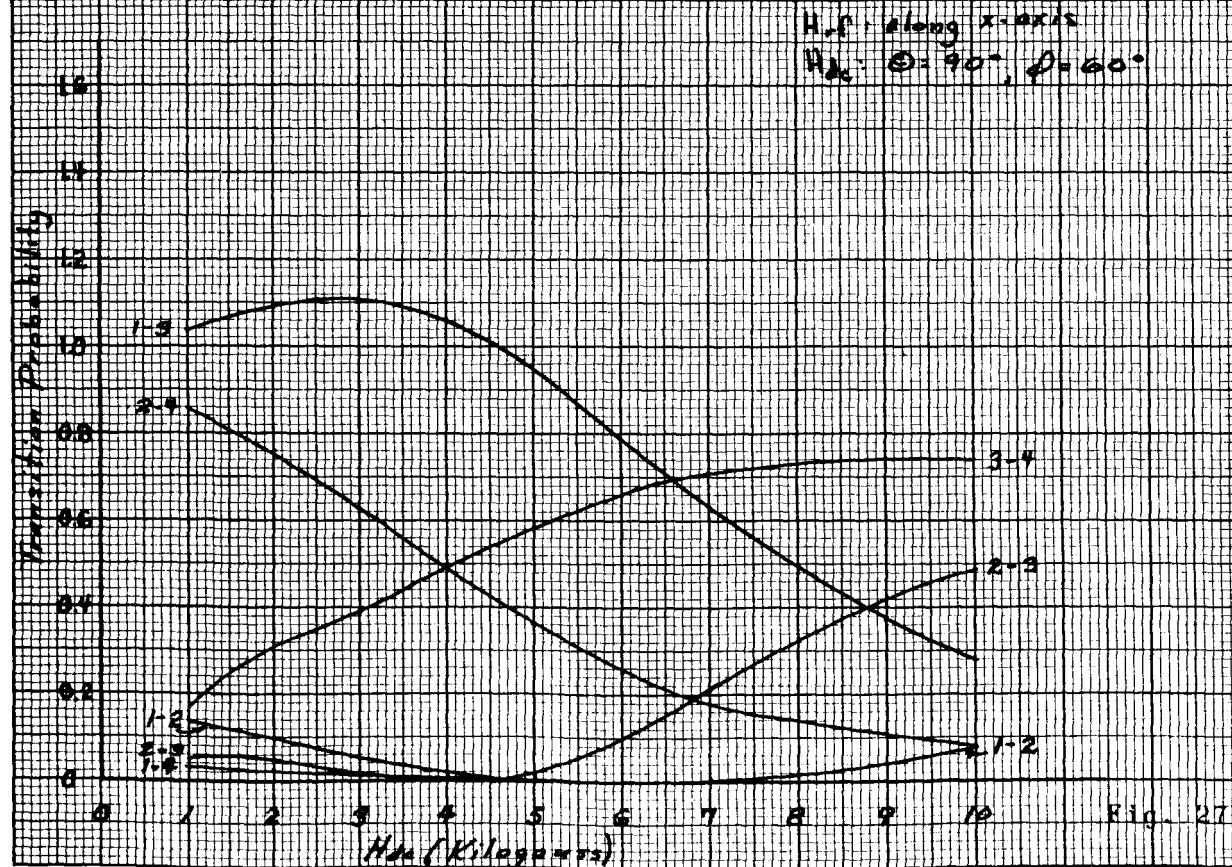
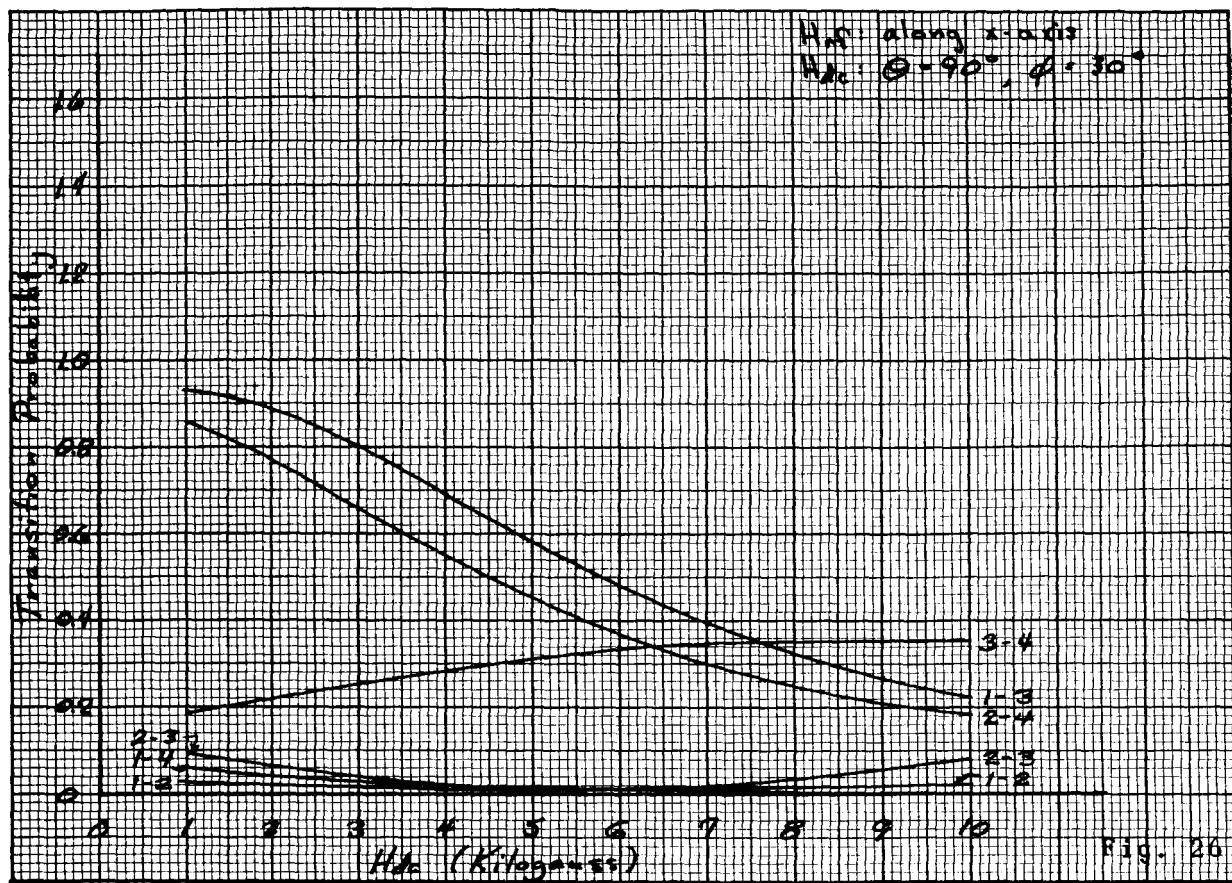
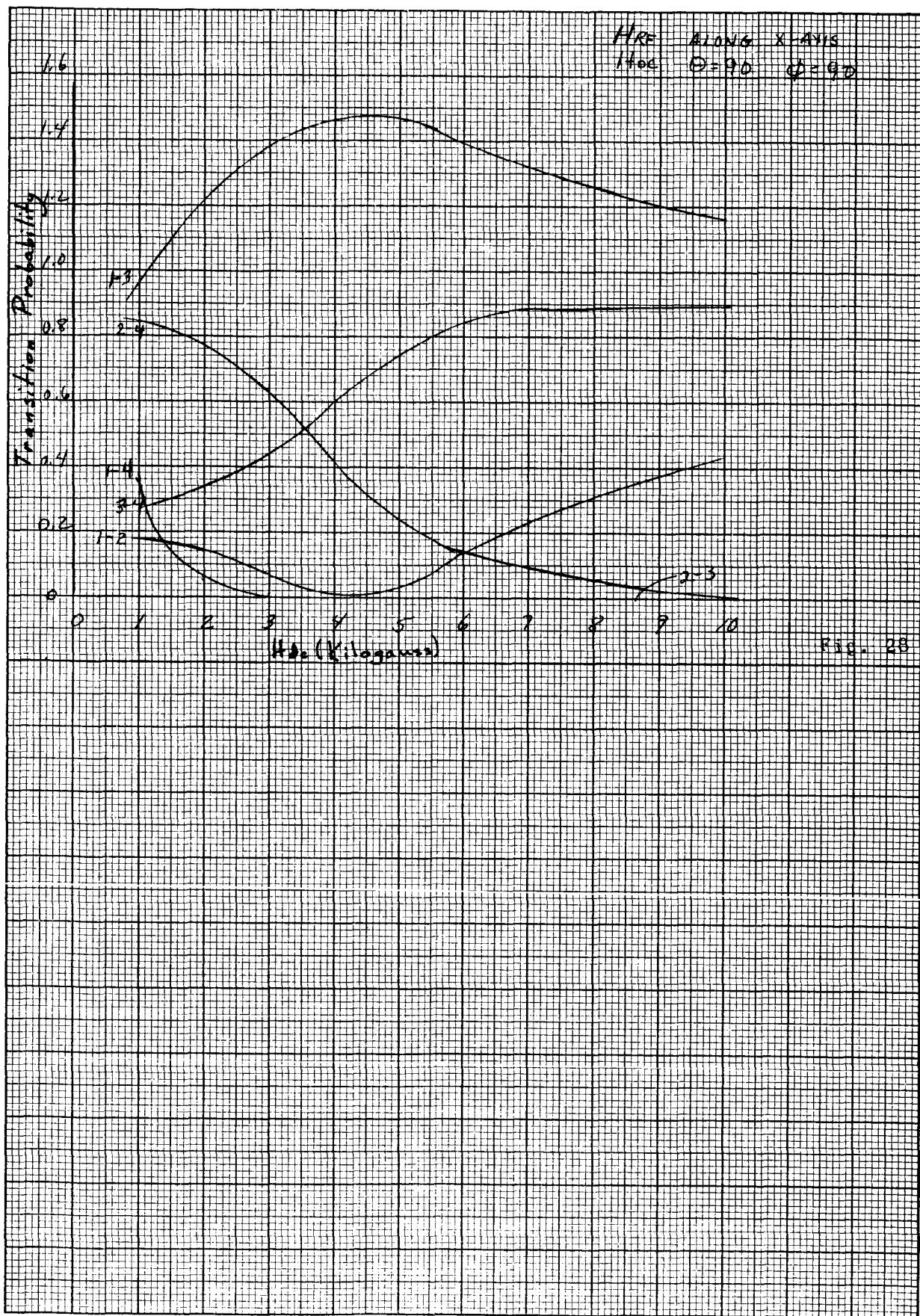


FIG. 21









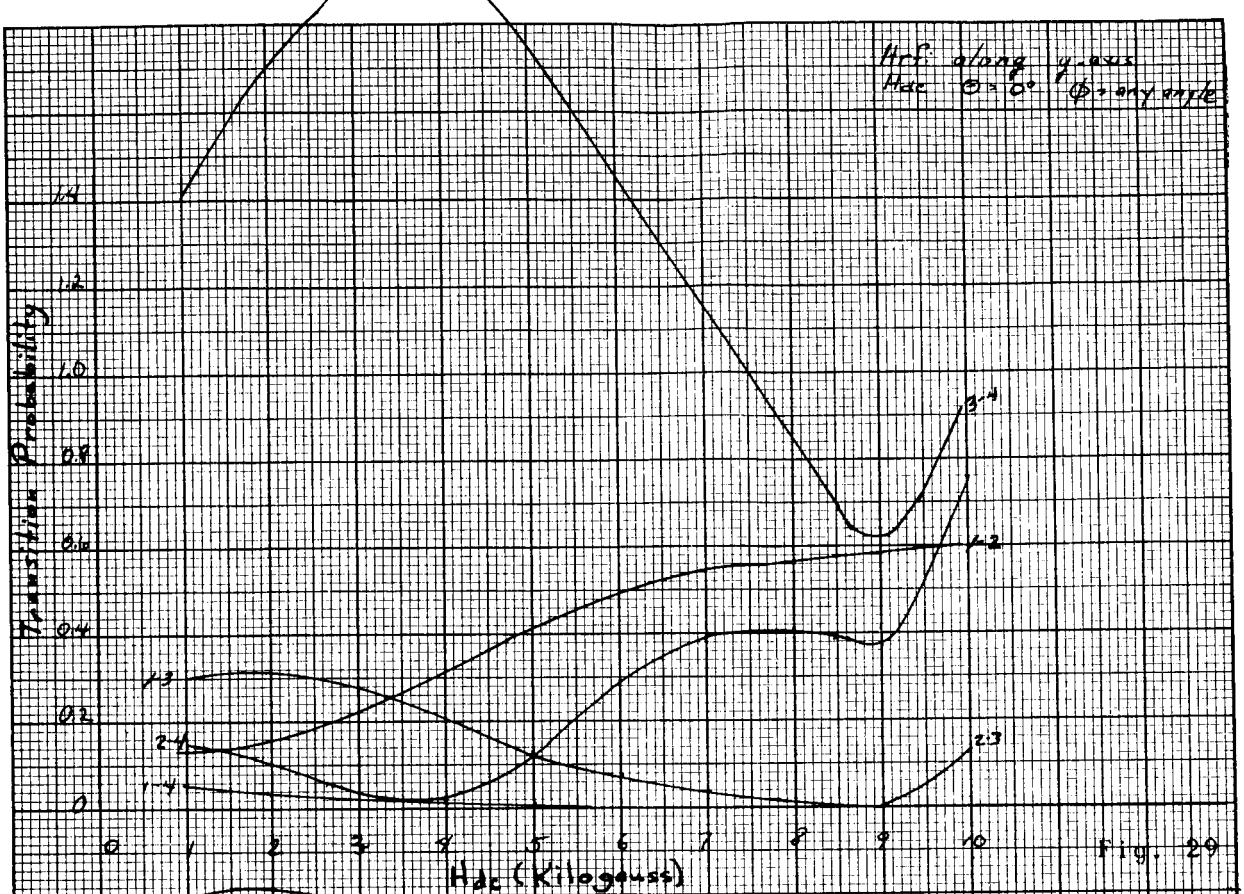


fig. 29

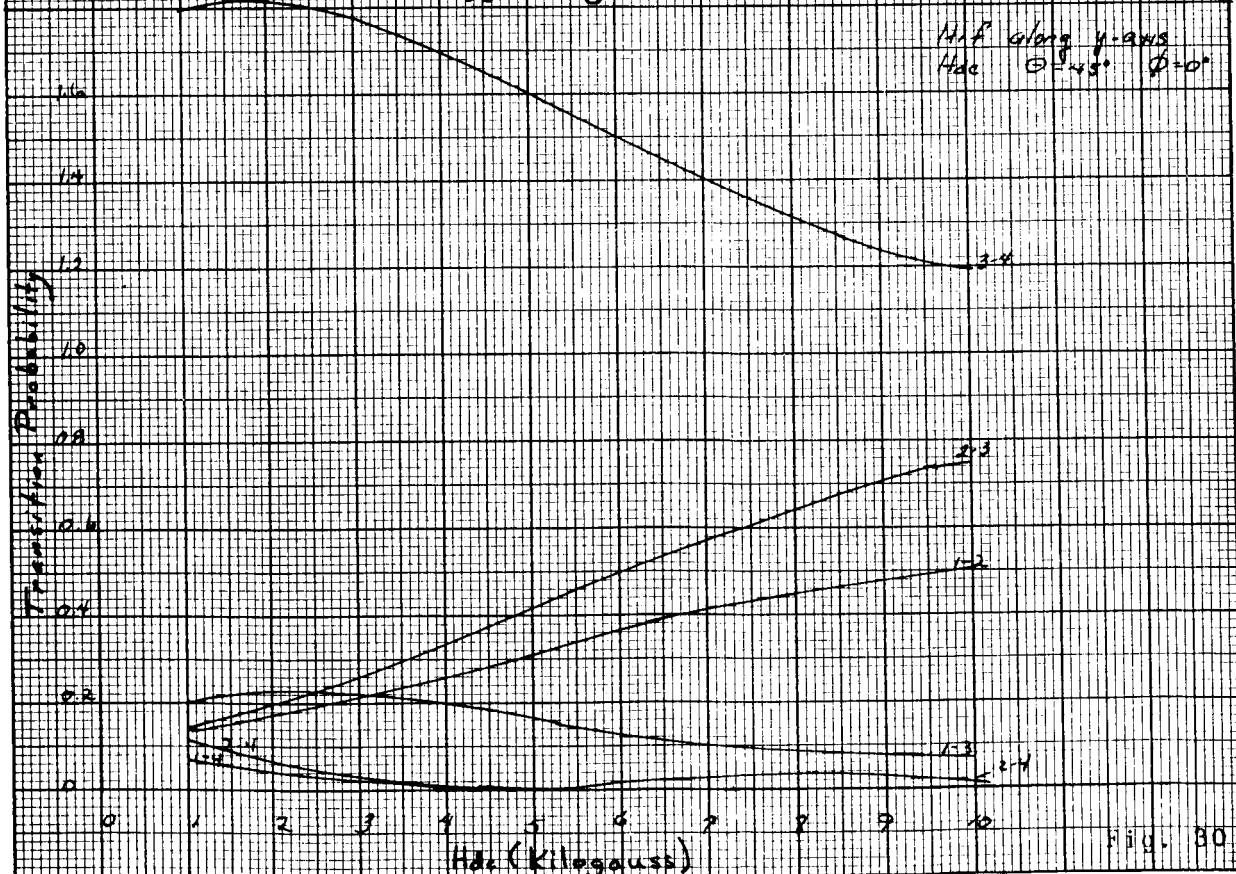
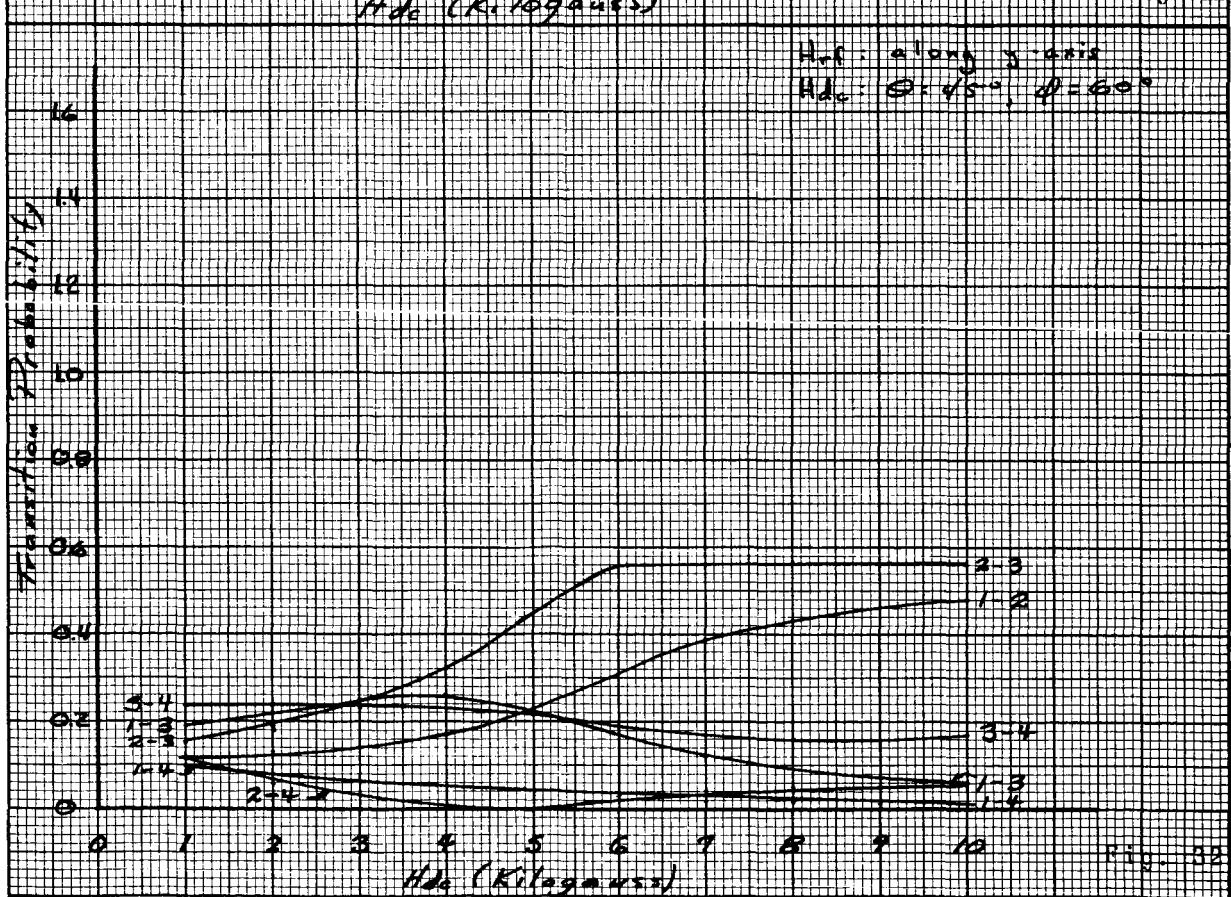
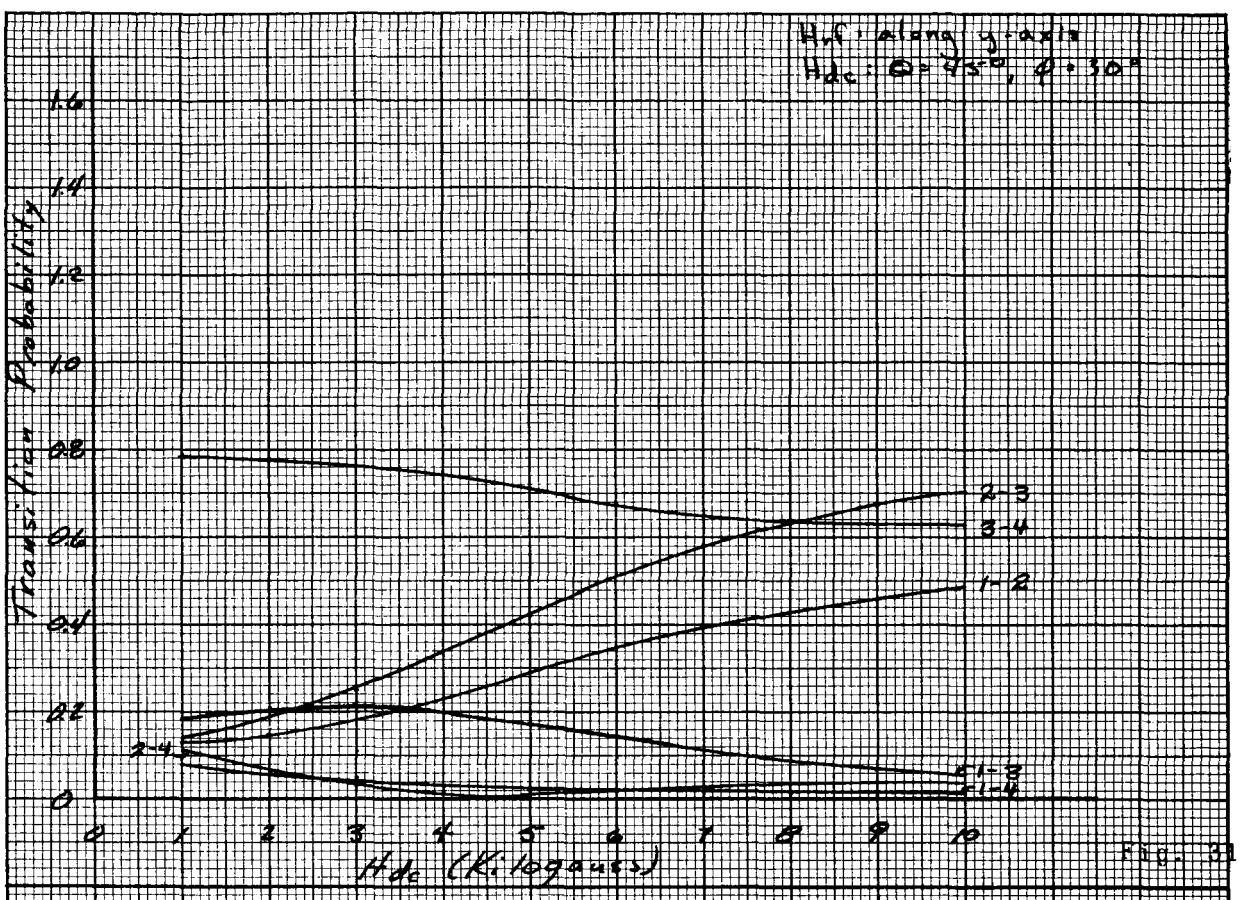
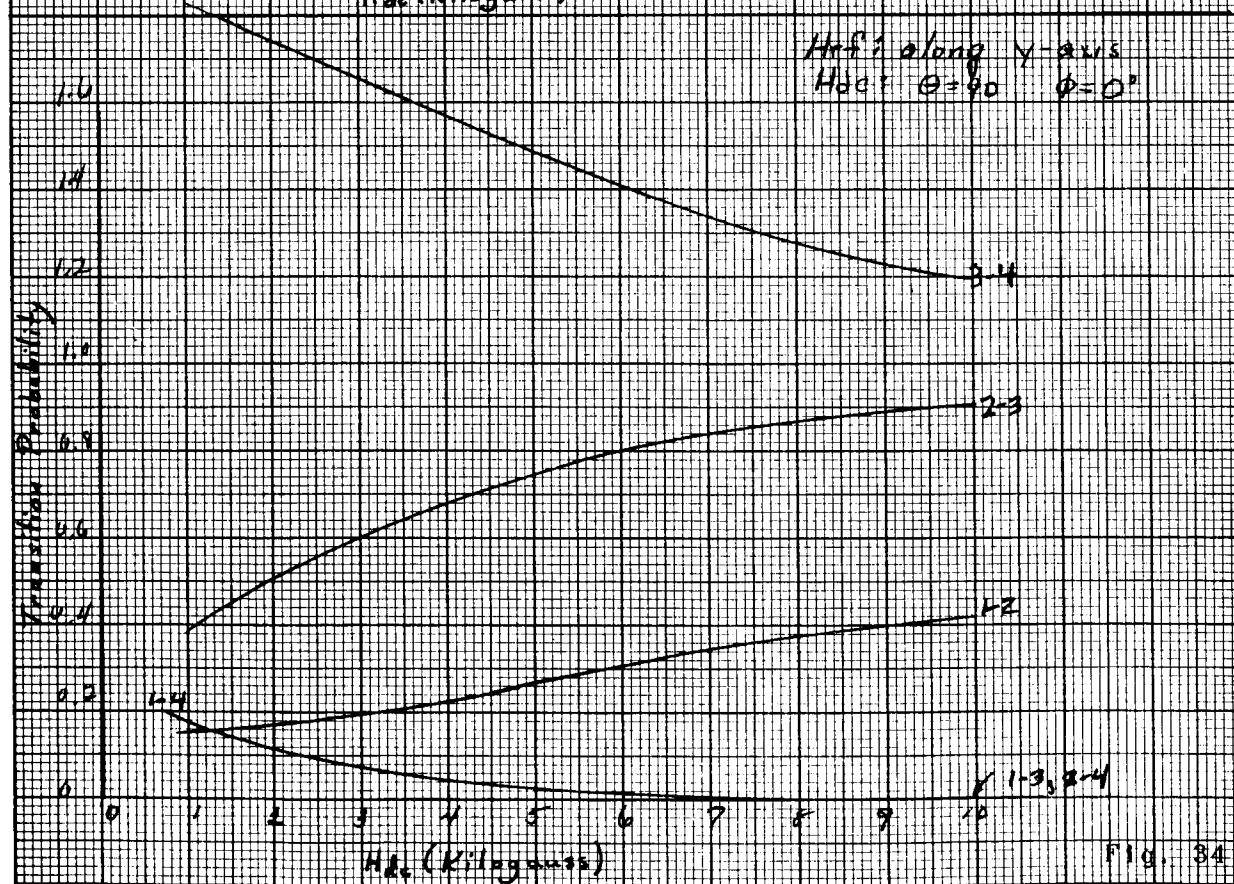
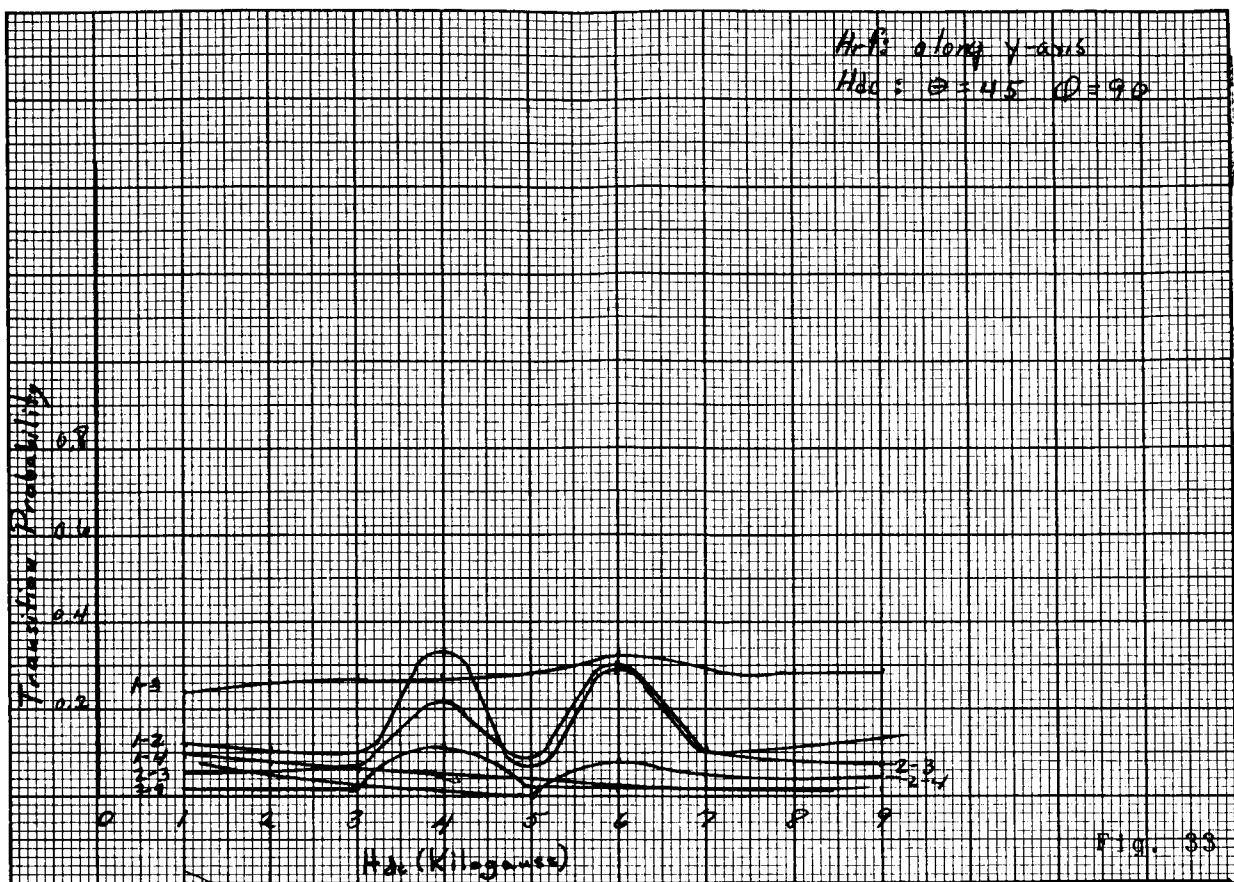
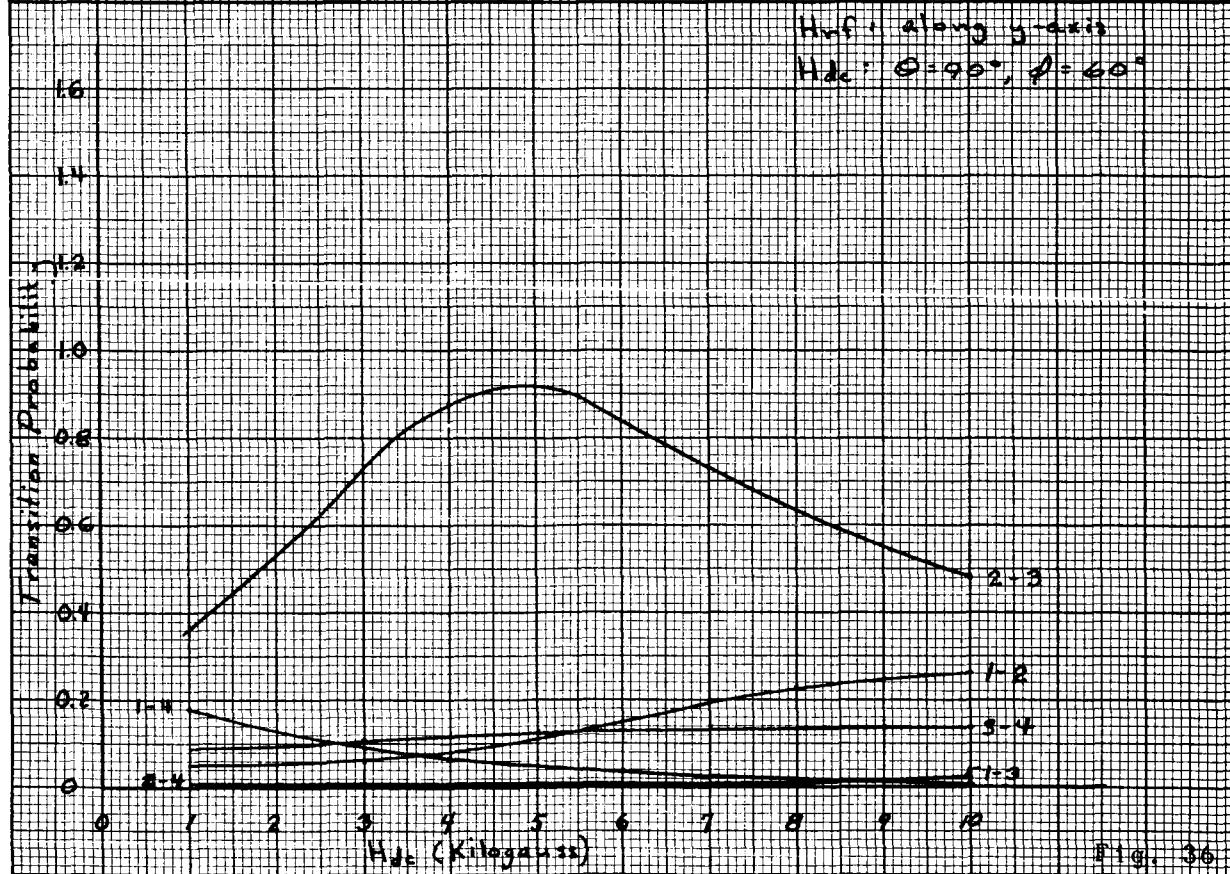
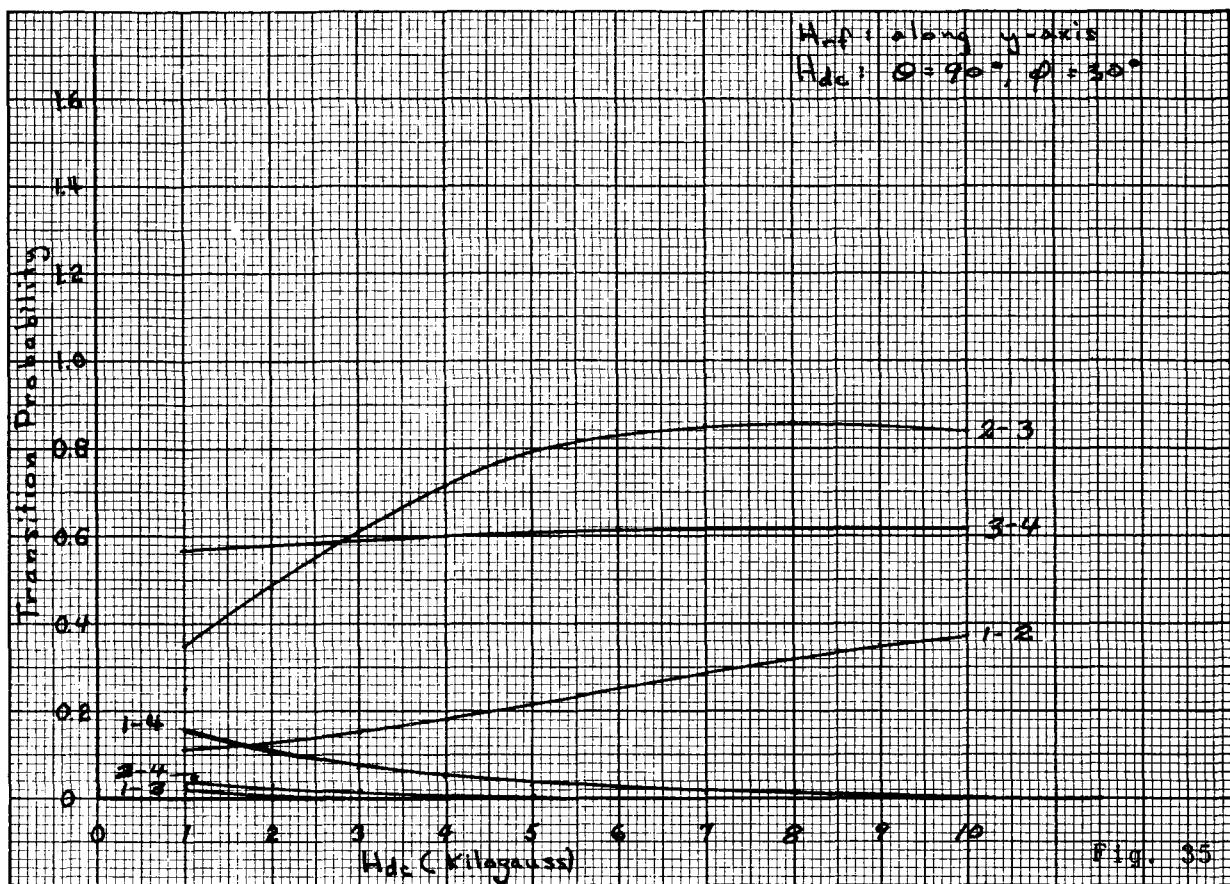
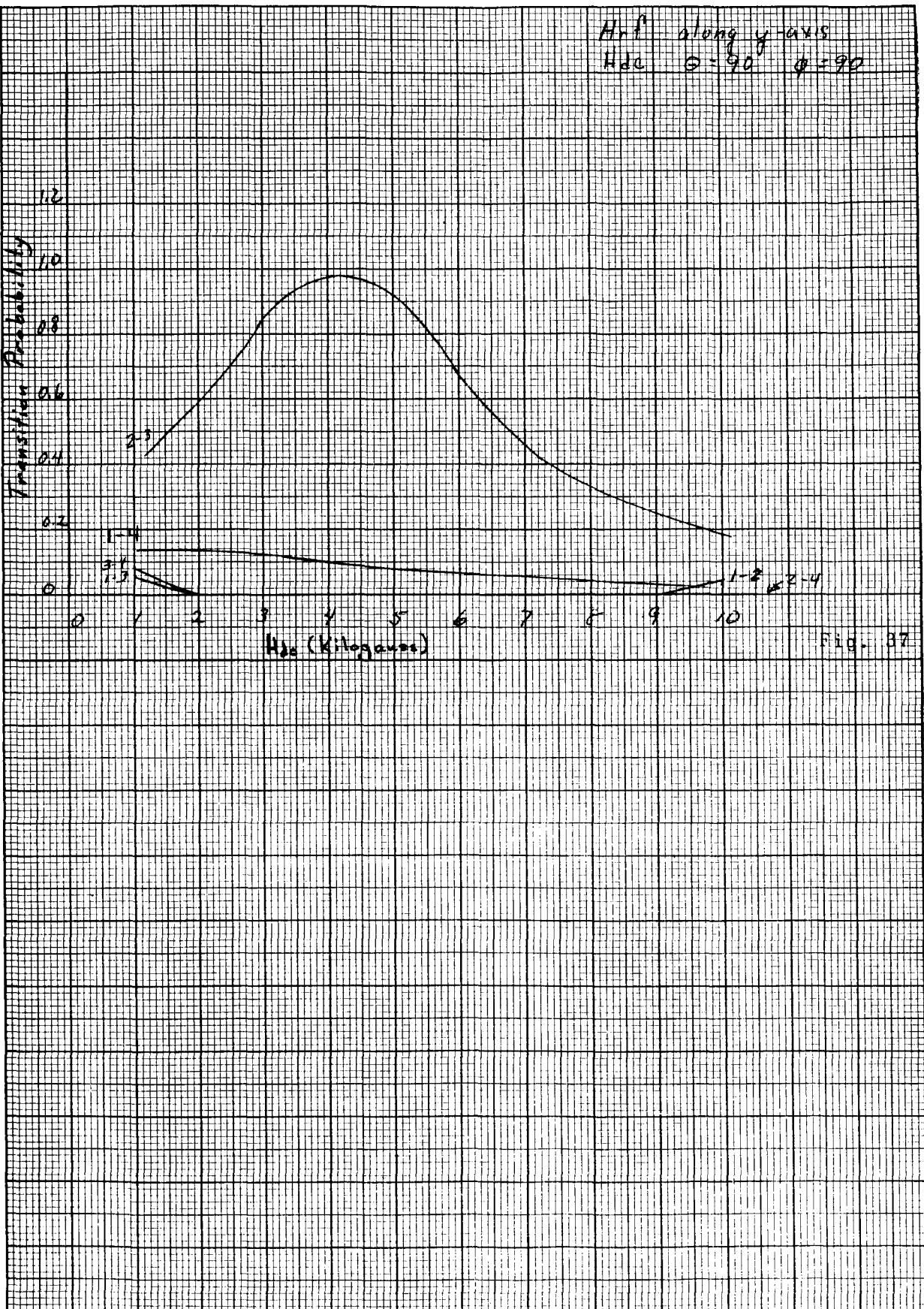


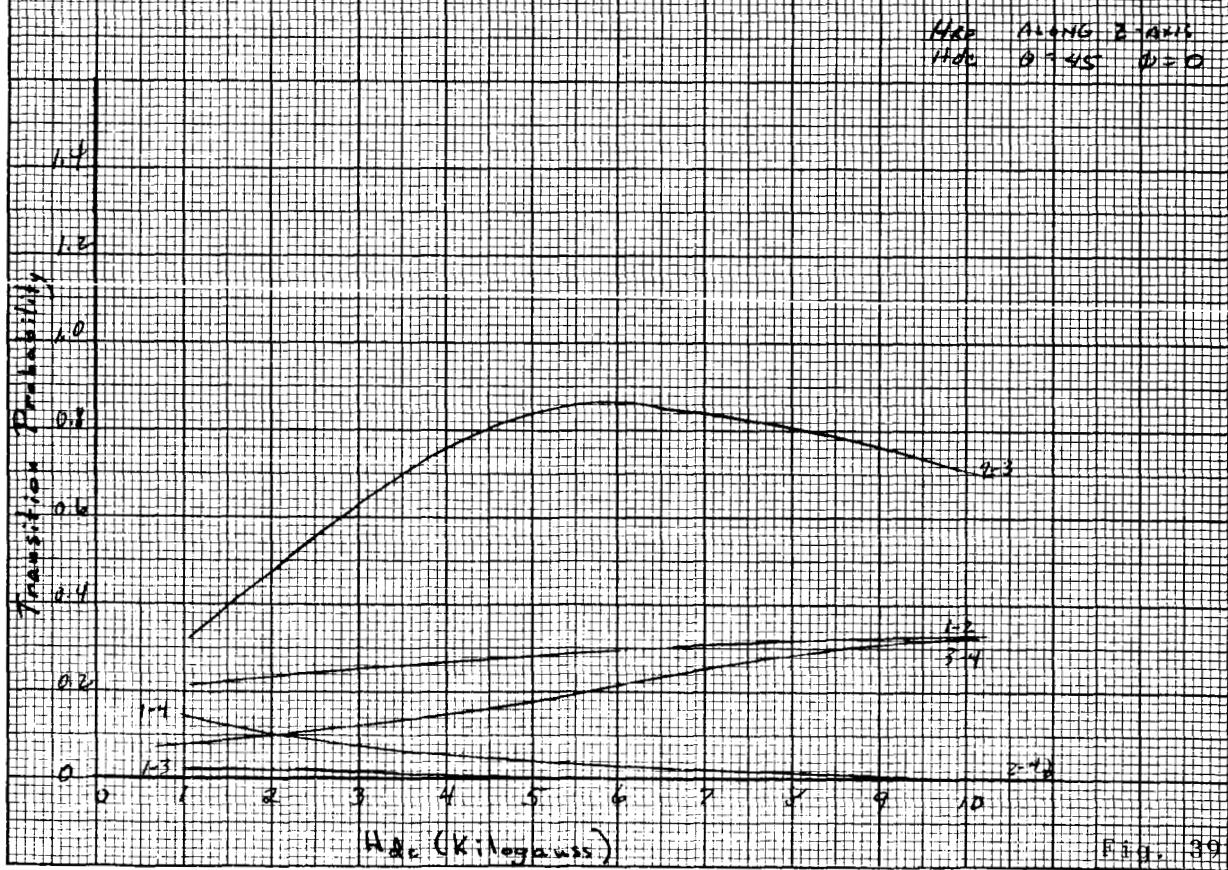
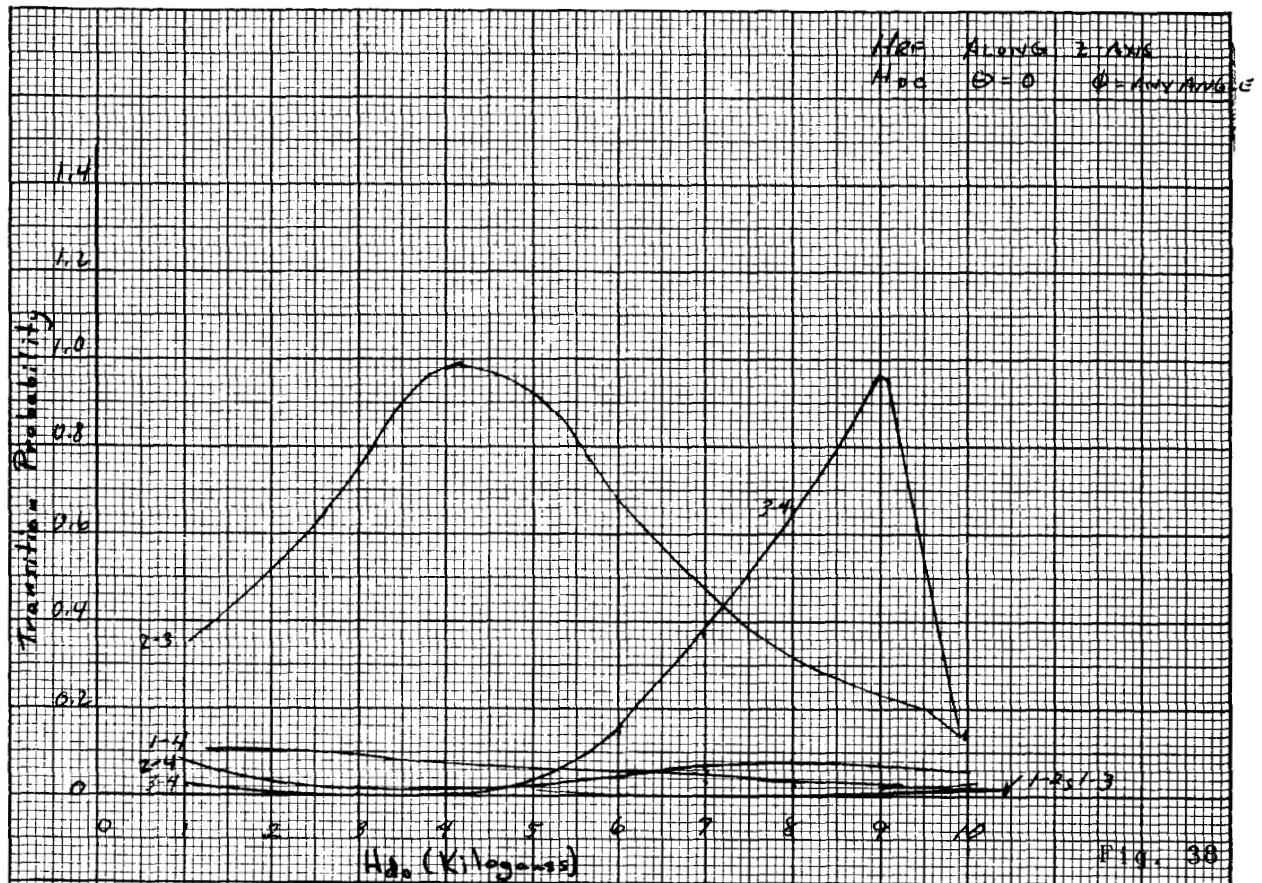
fig. 30

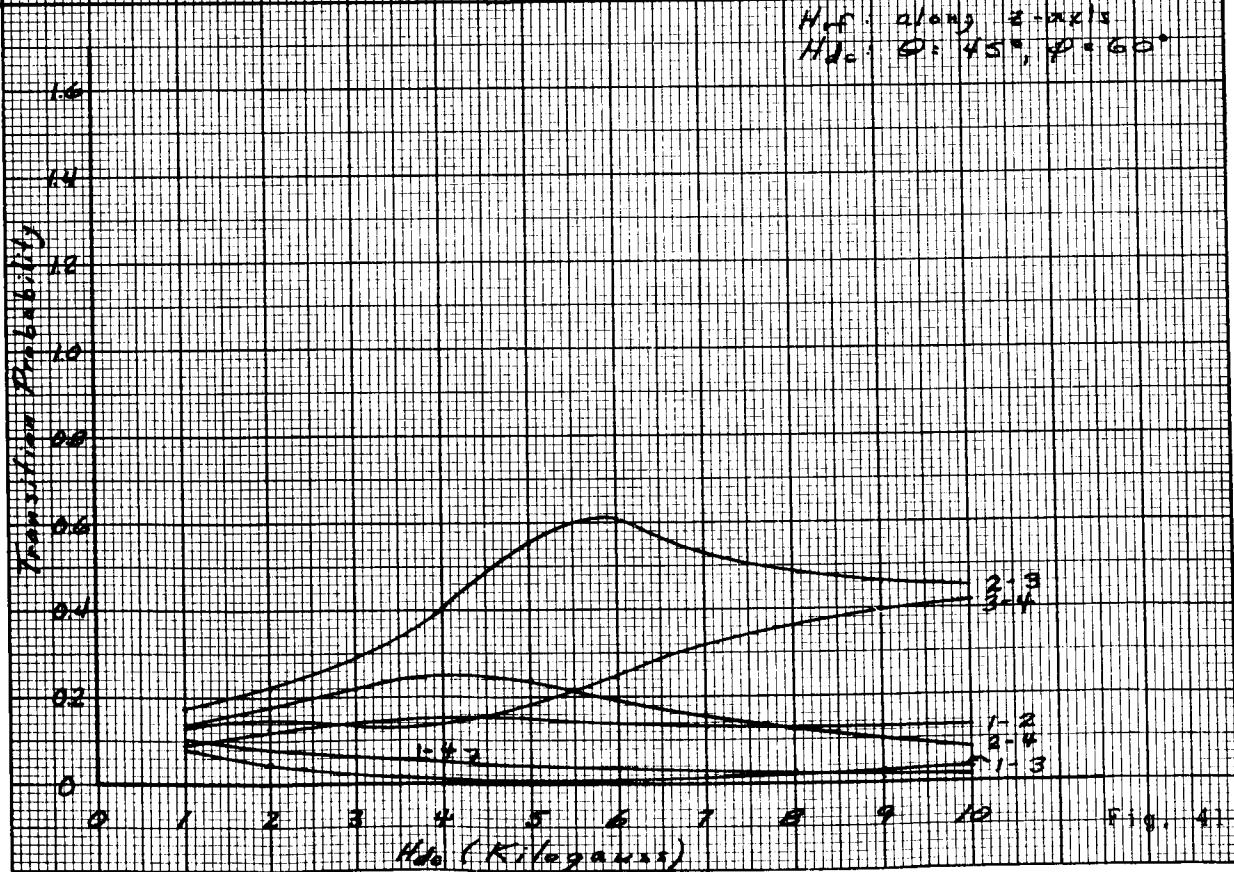
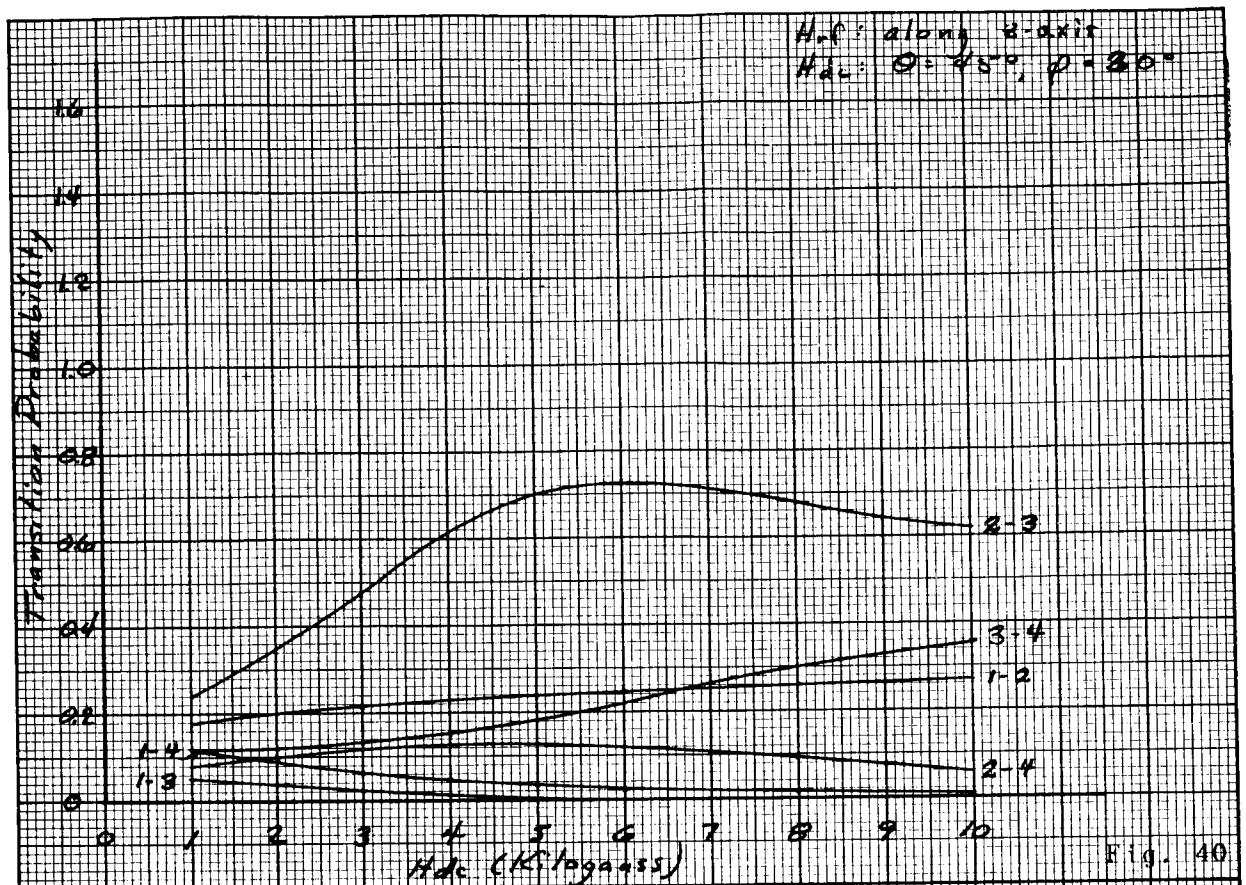


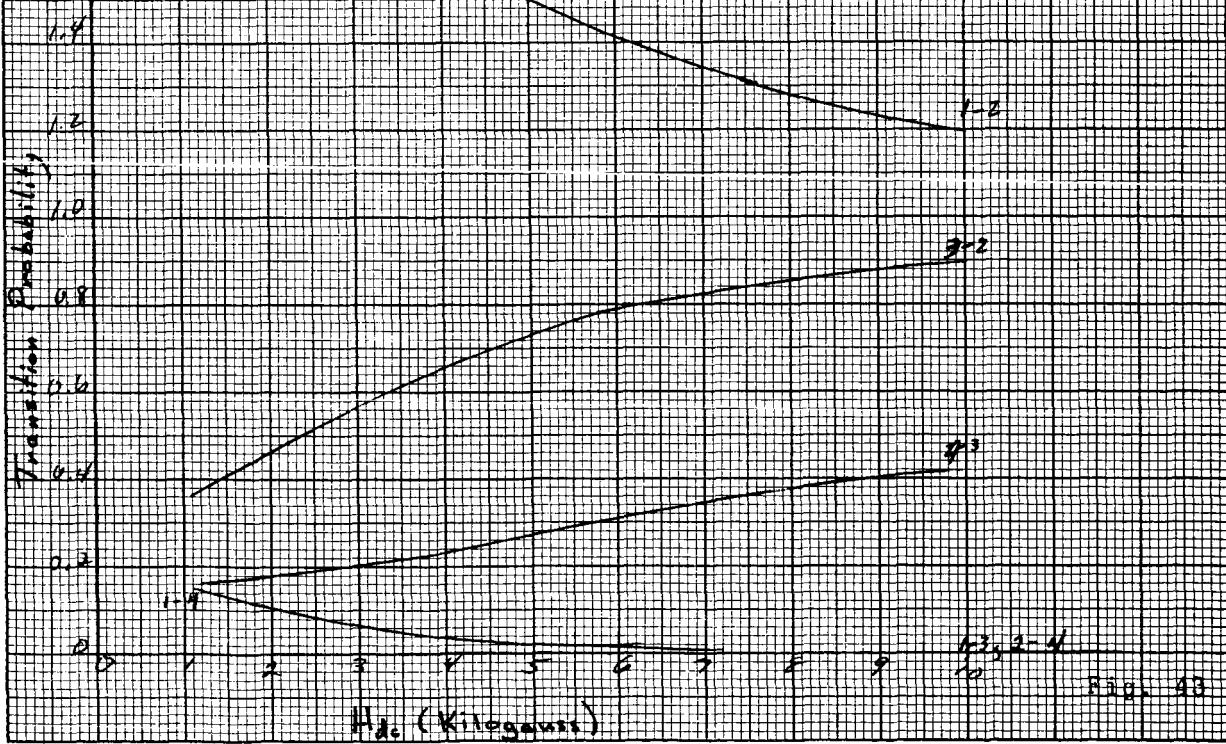
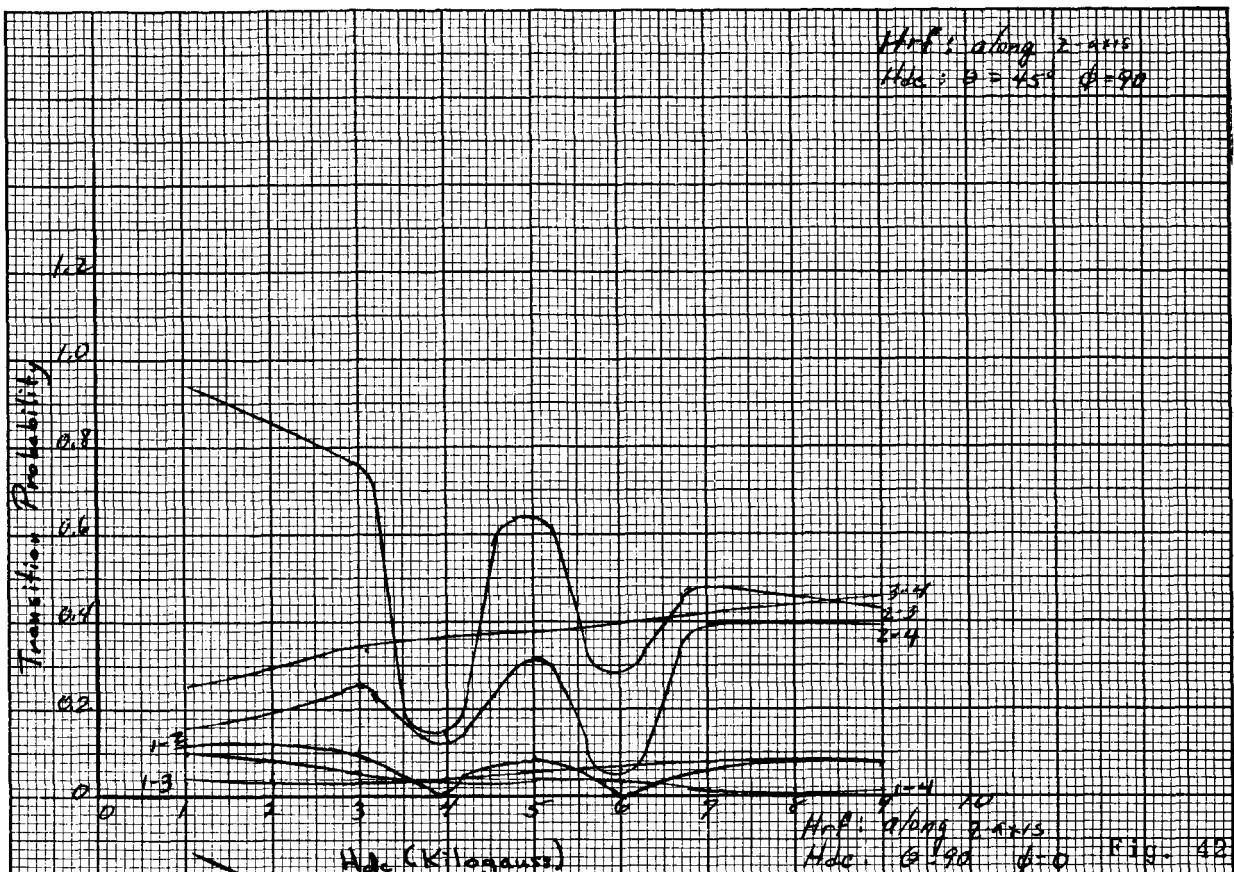












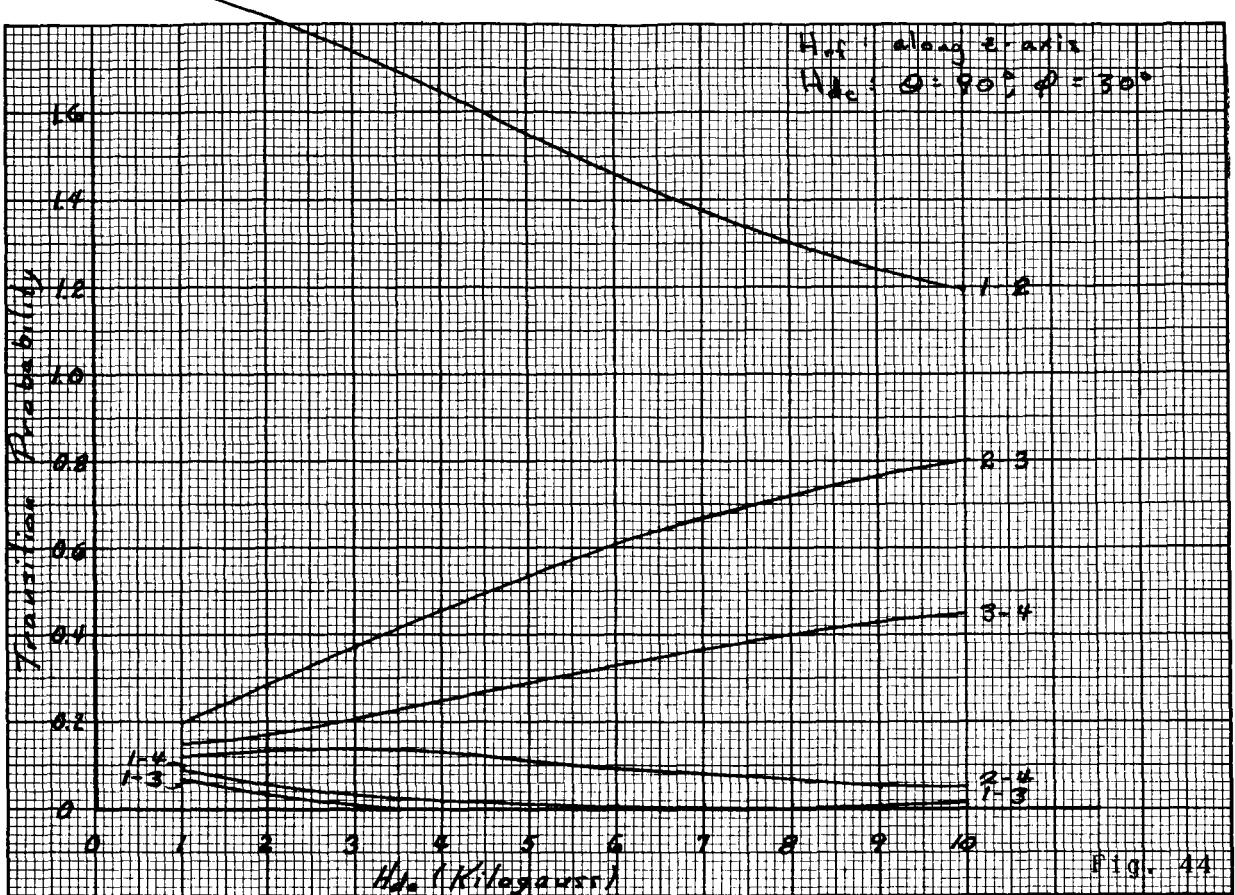


FIG. 44

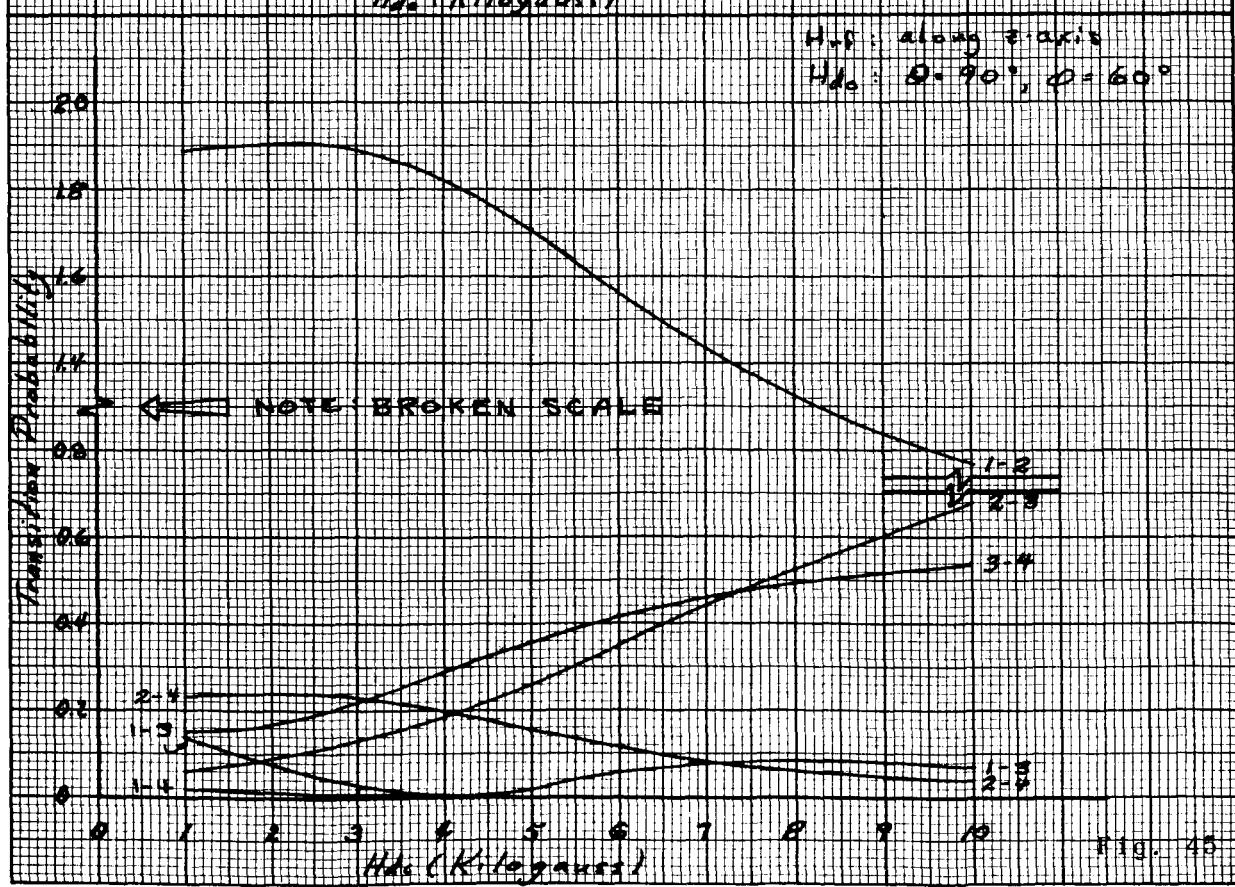
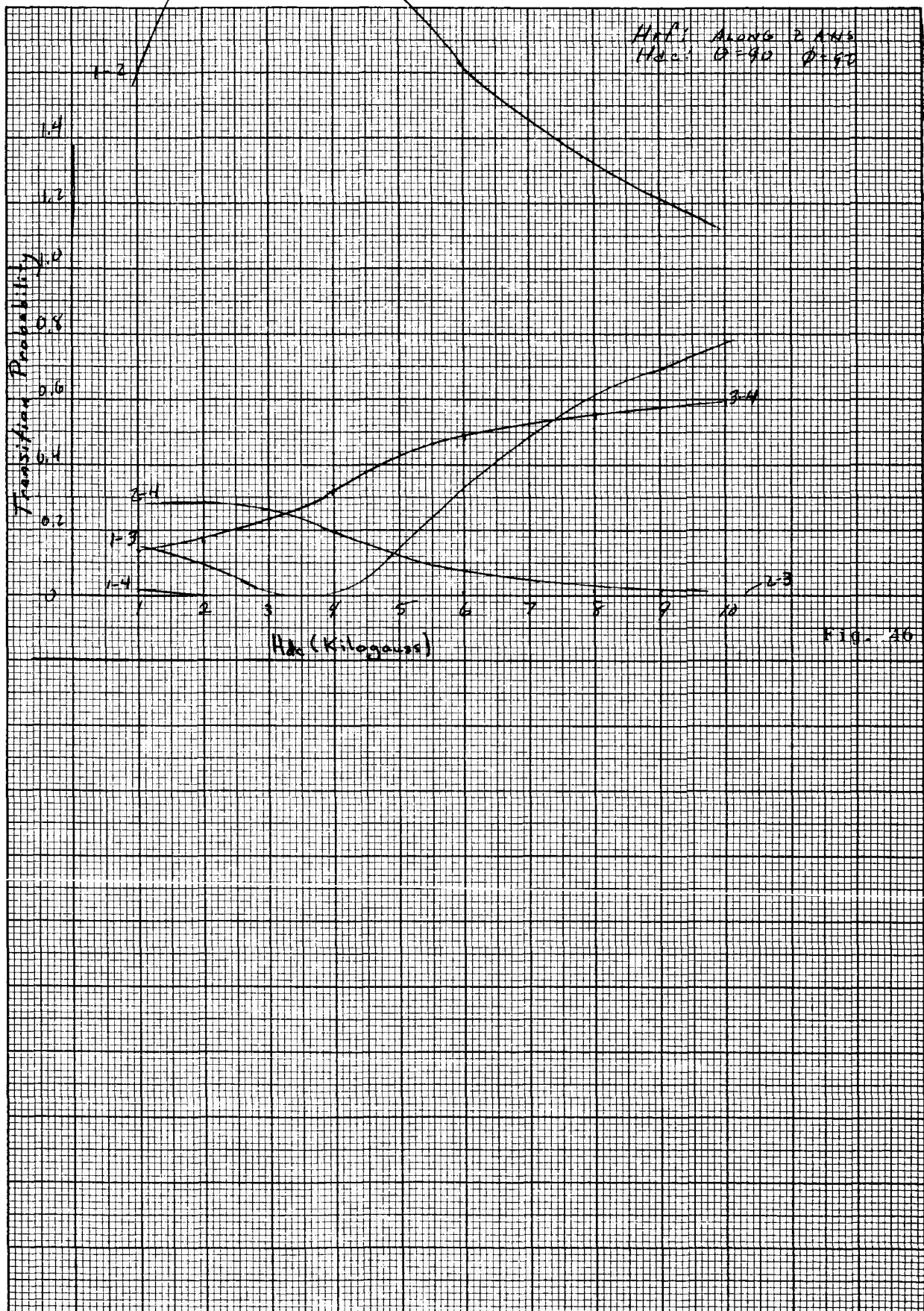


FIG. 45



Figs. 47-73

$|s_{x_{ij}}|^2$, $|s_{y_{ij}}|^2$ and $|s_{z_{ij}}|^2$ for $\text{Cr}^{3+}:\text{ZnWO}_4$

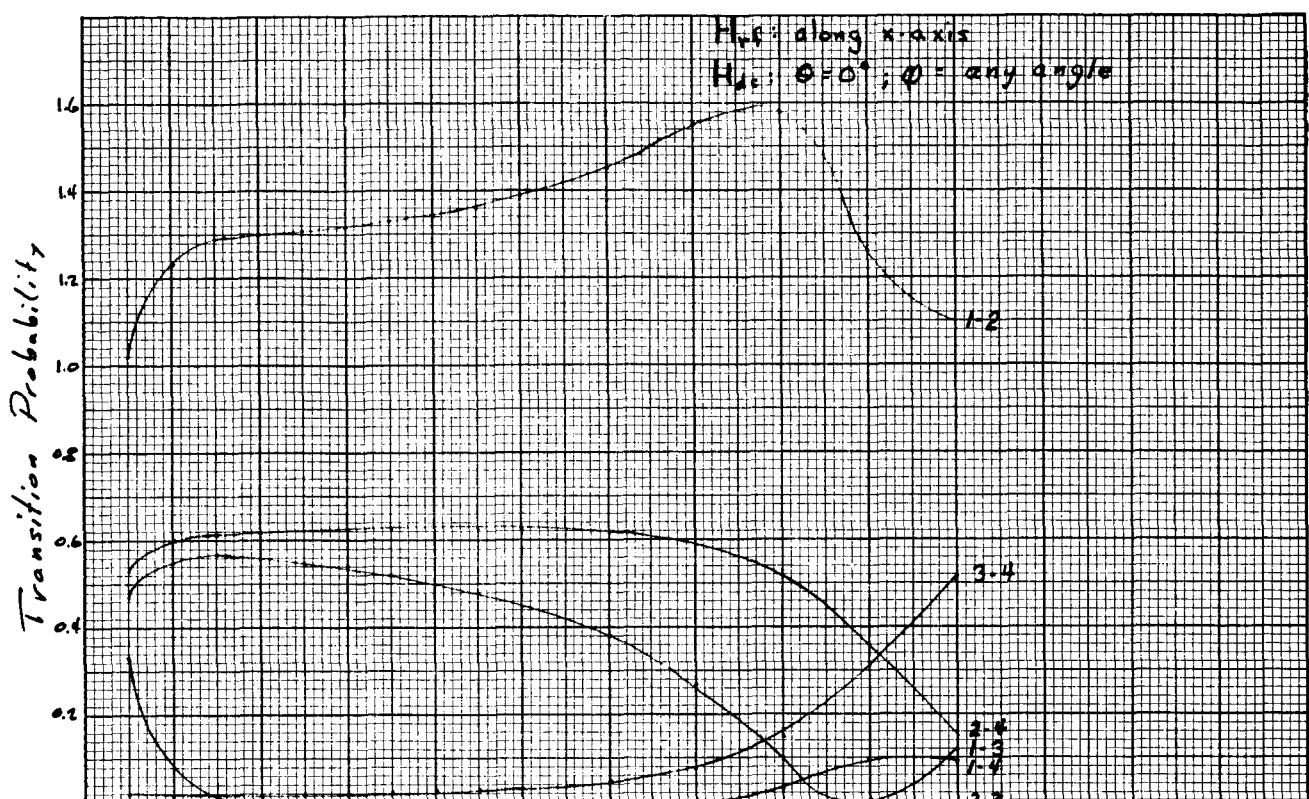


FIG. 47

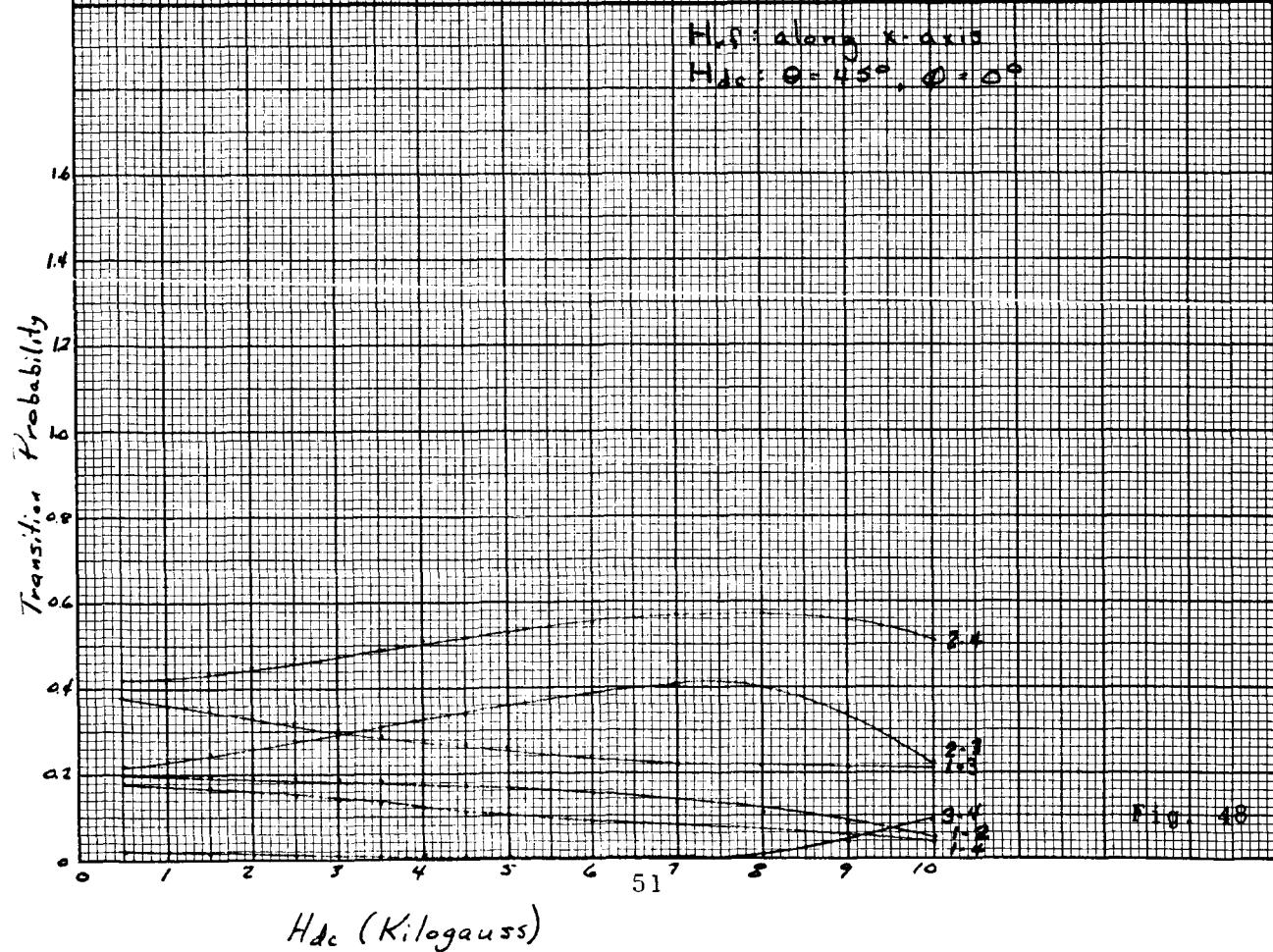


FIG. 48

H_{dc} (Kilogauss)

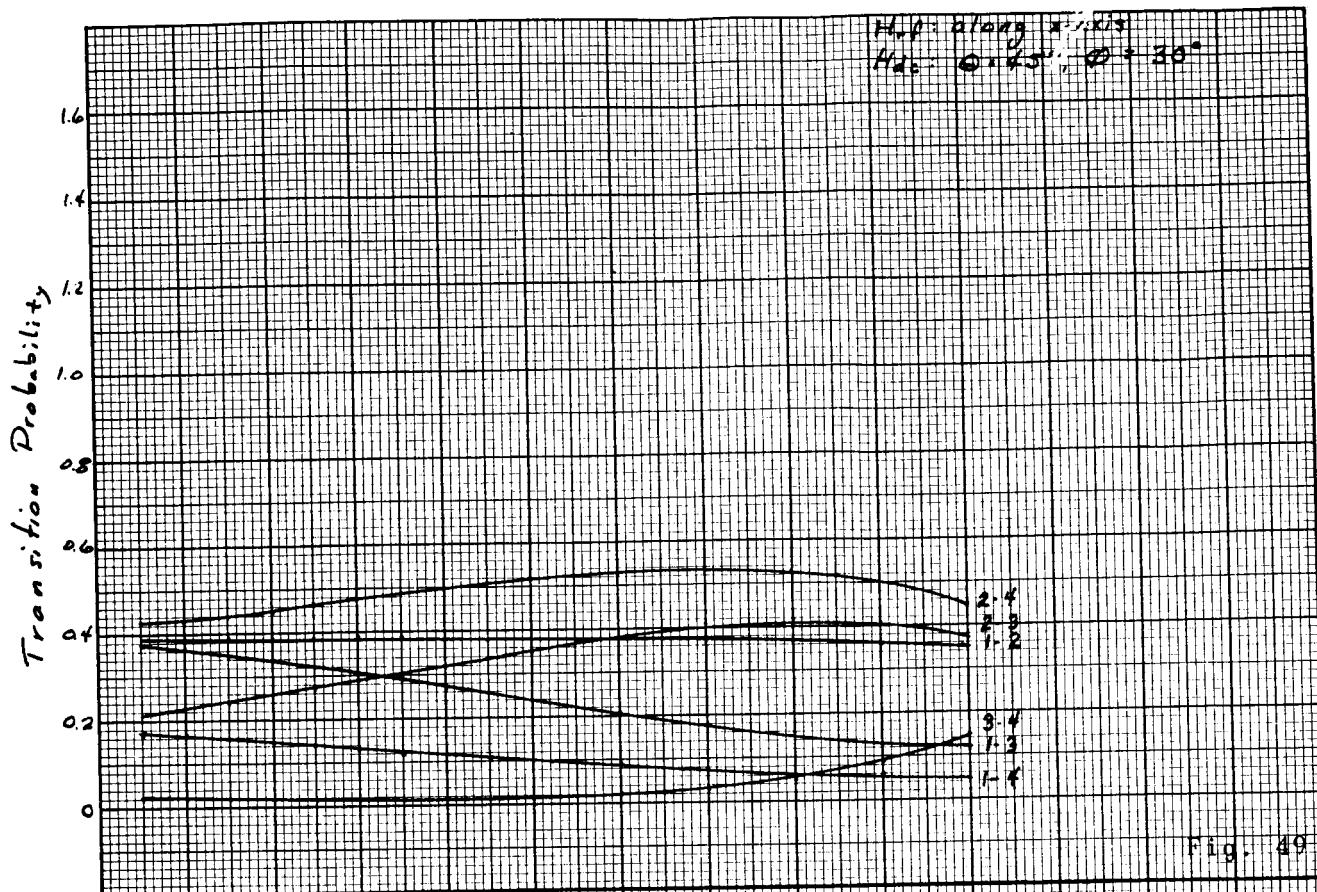


Fig. 49

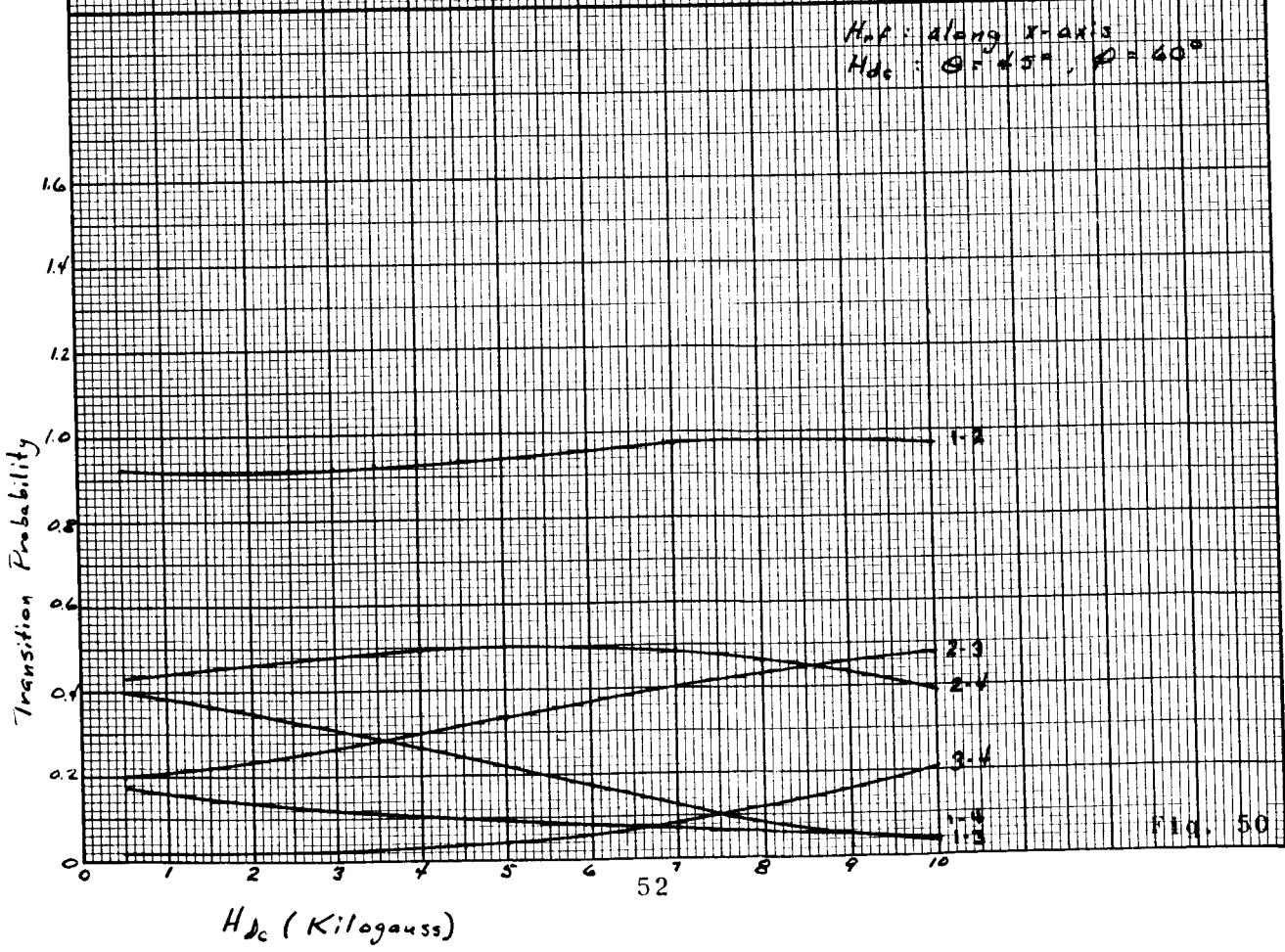
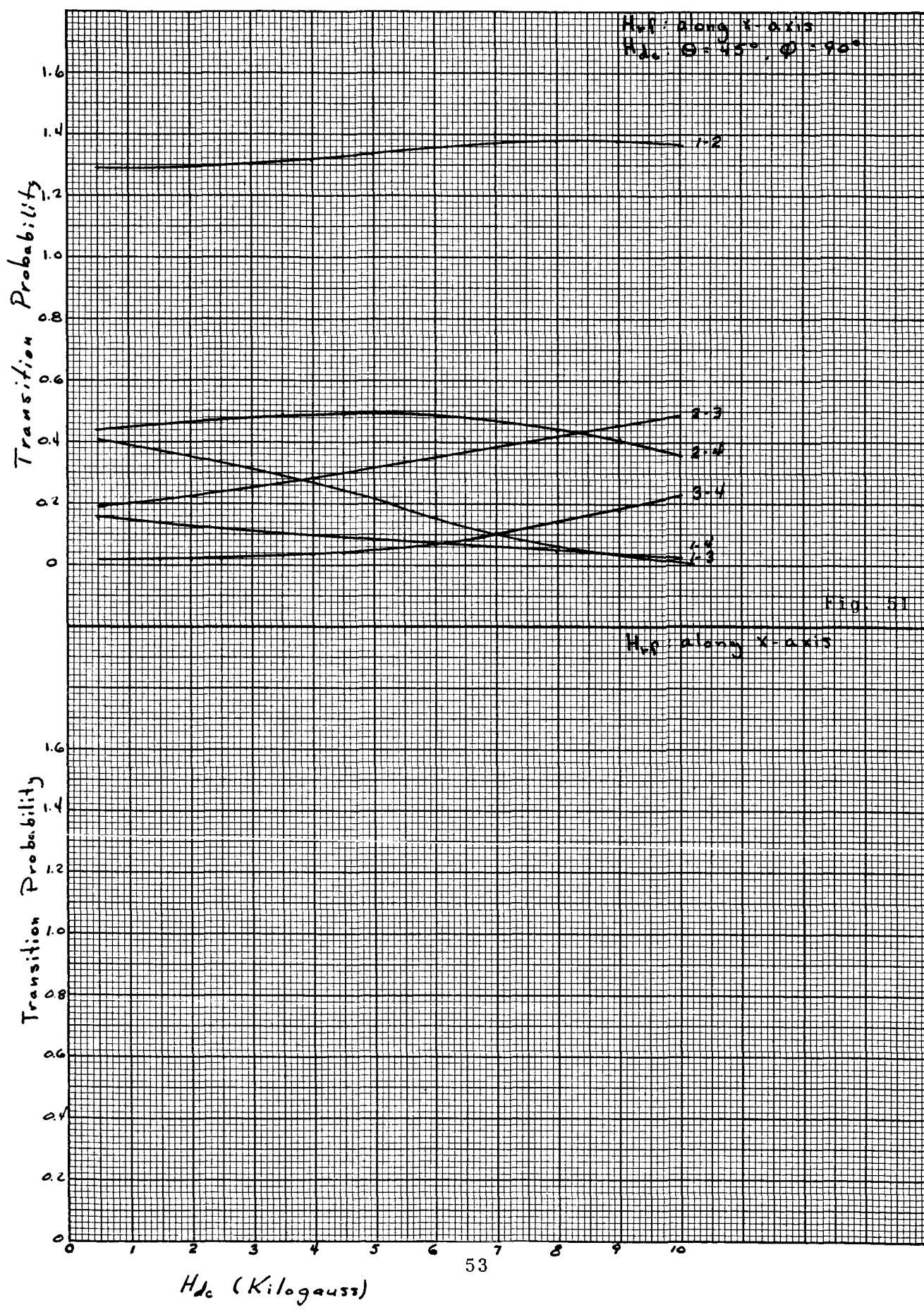
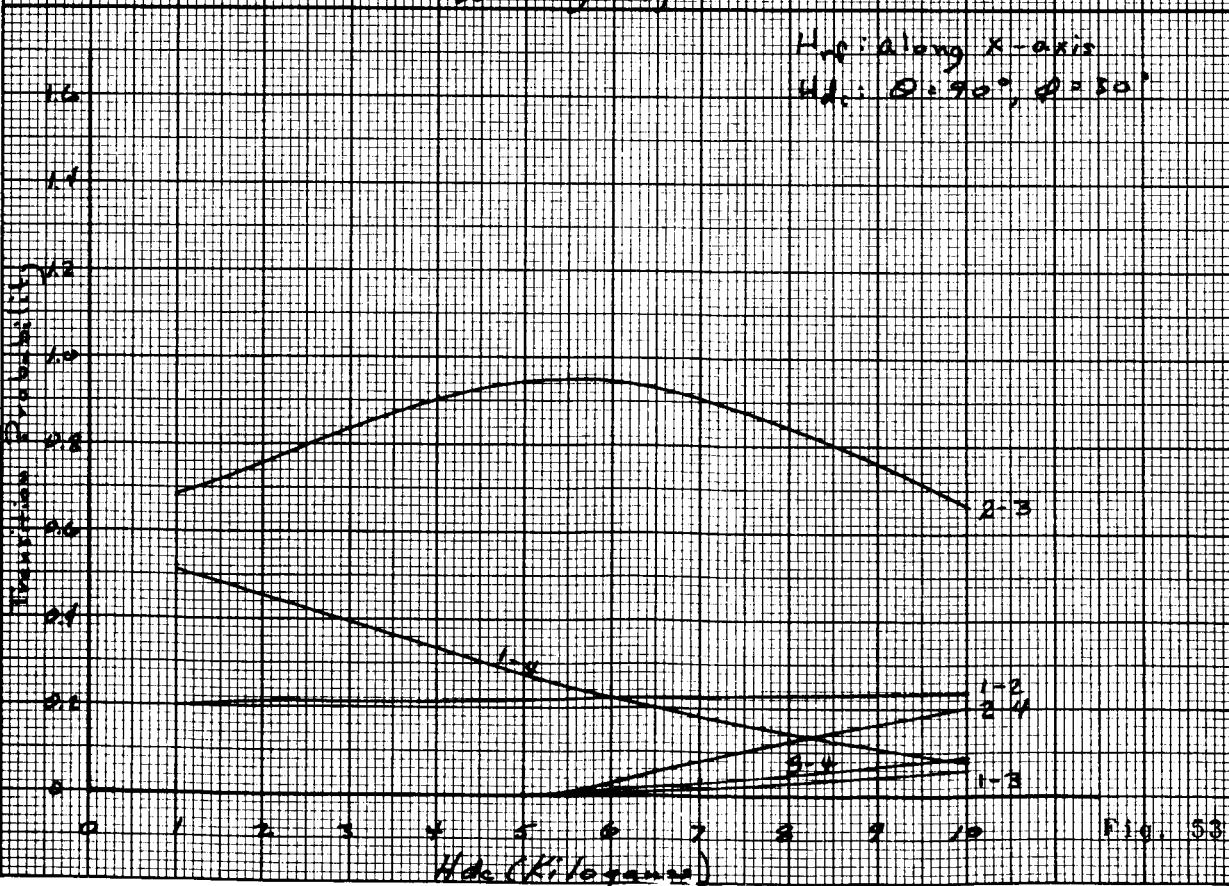
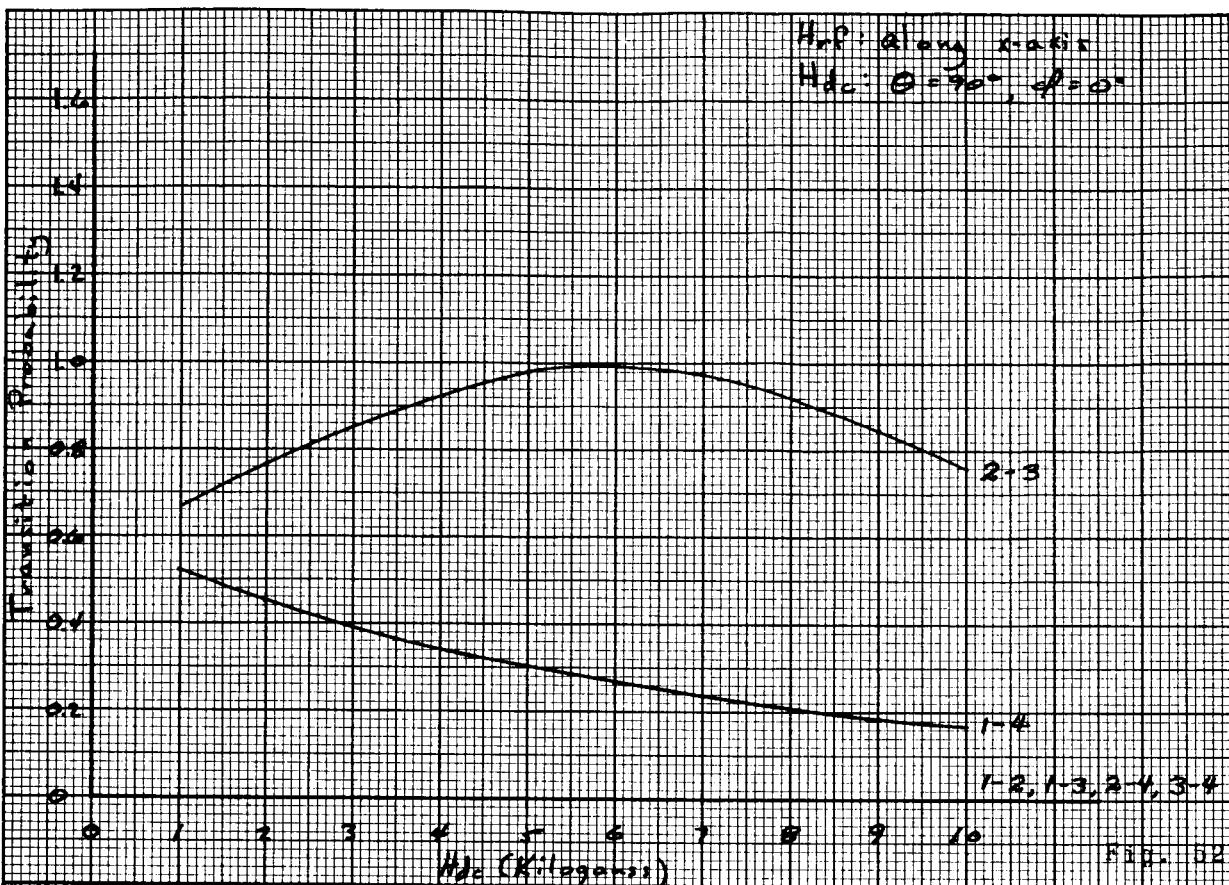
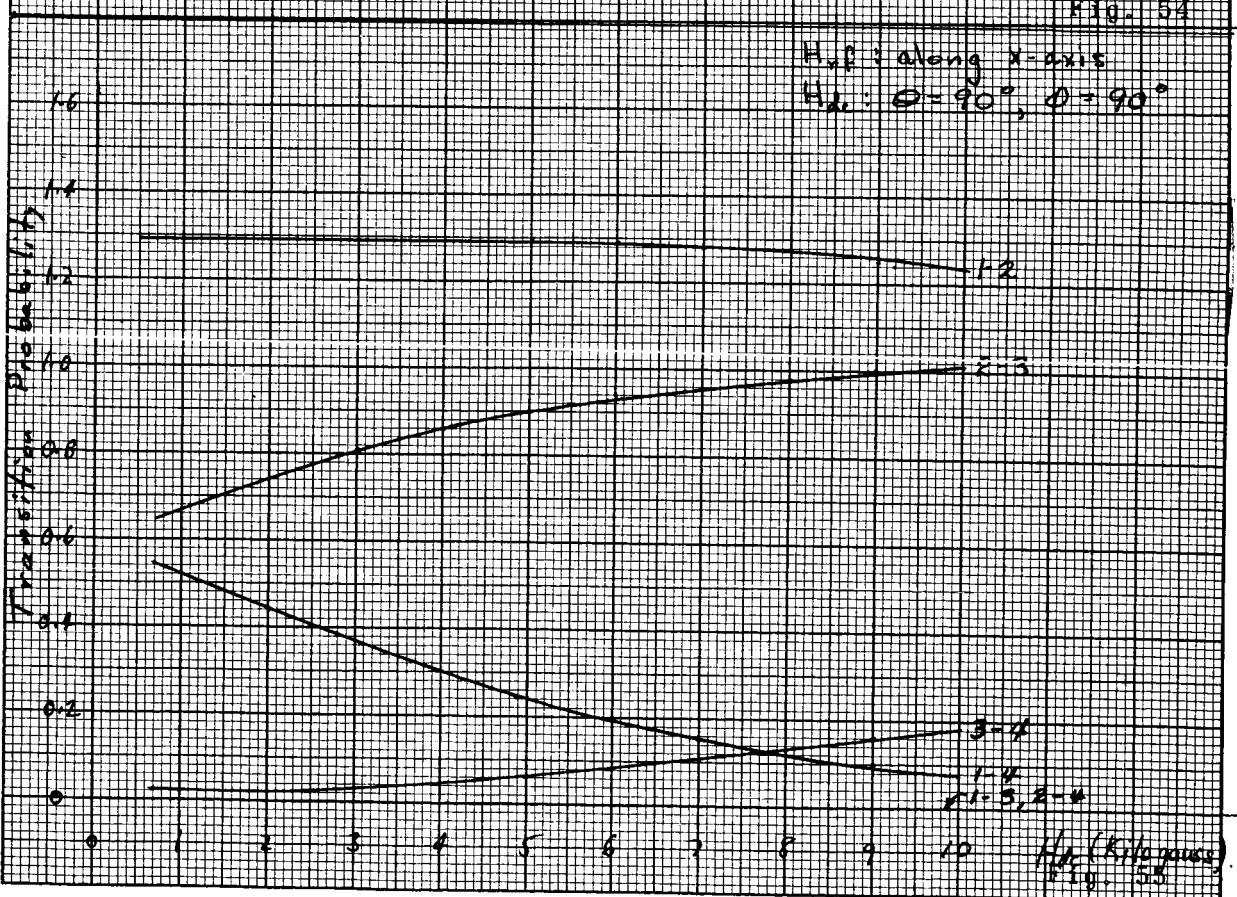
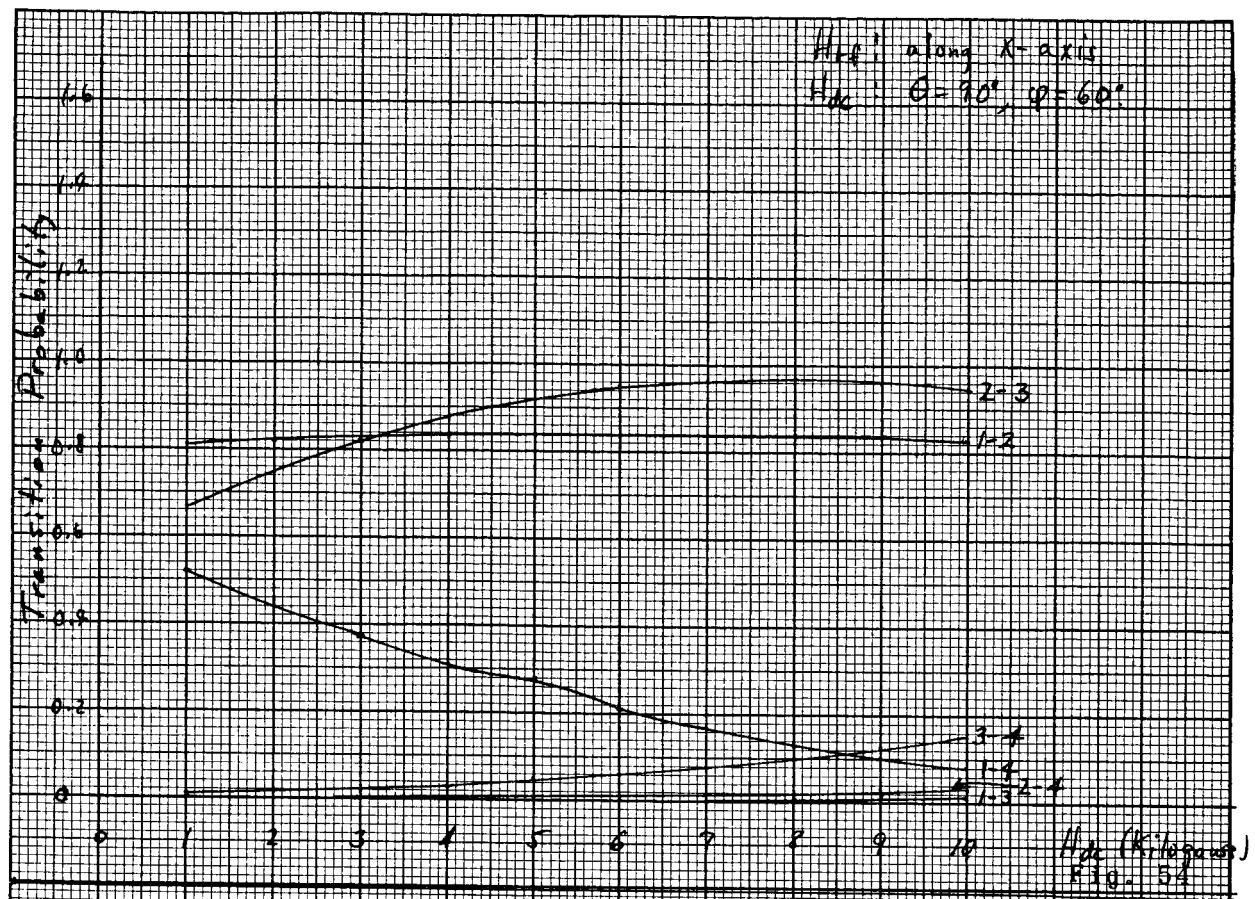


Fig. 50







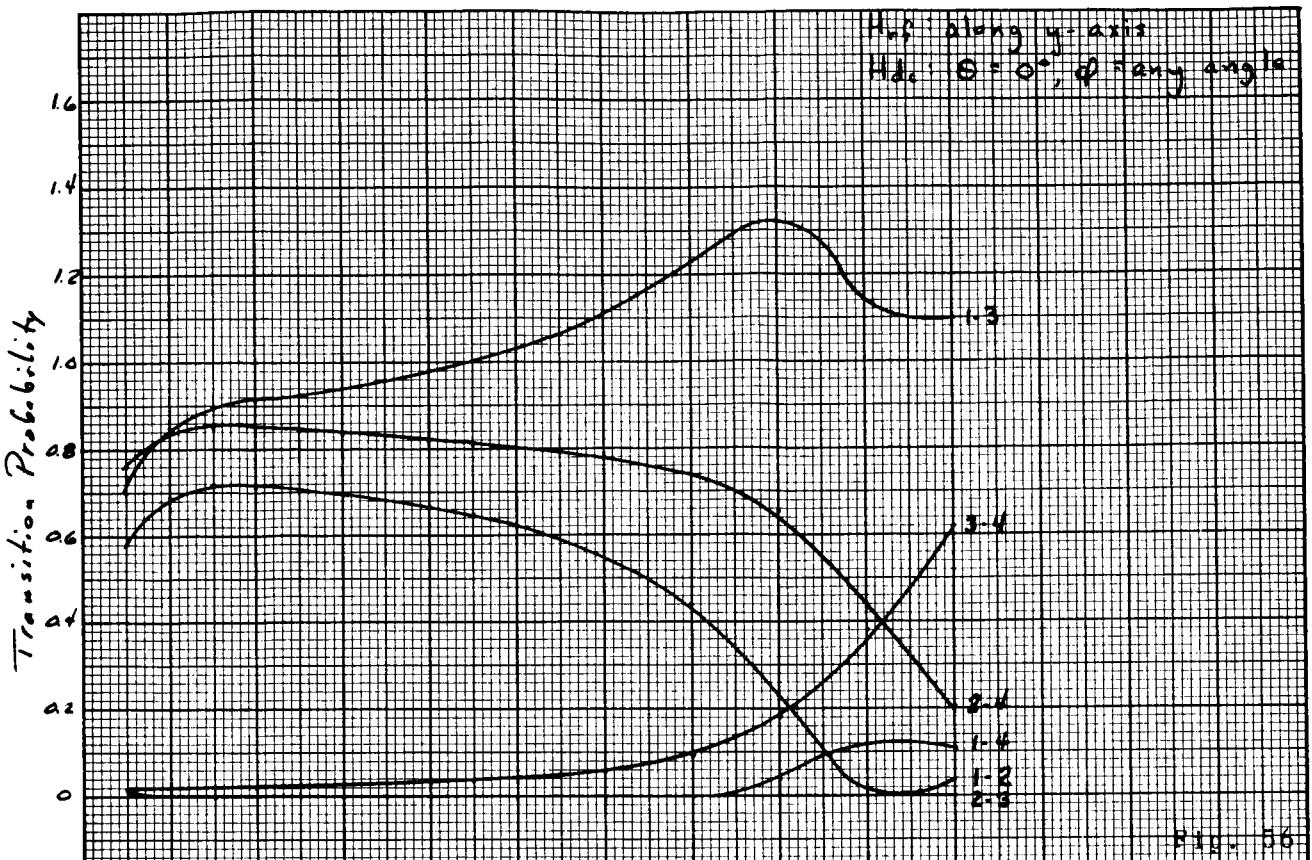


FIG. 56

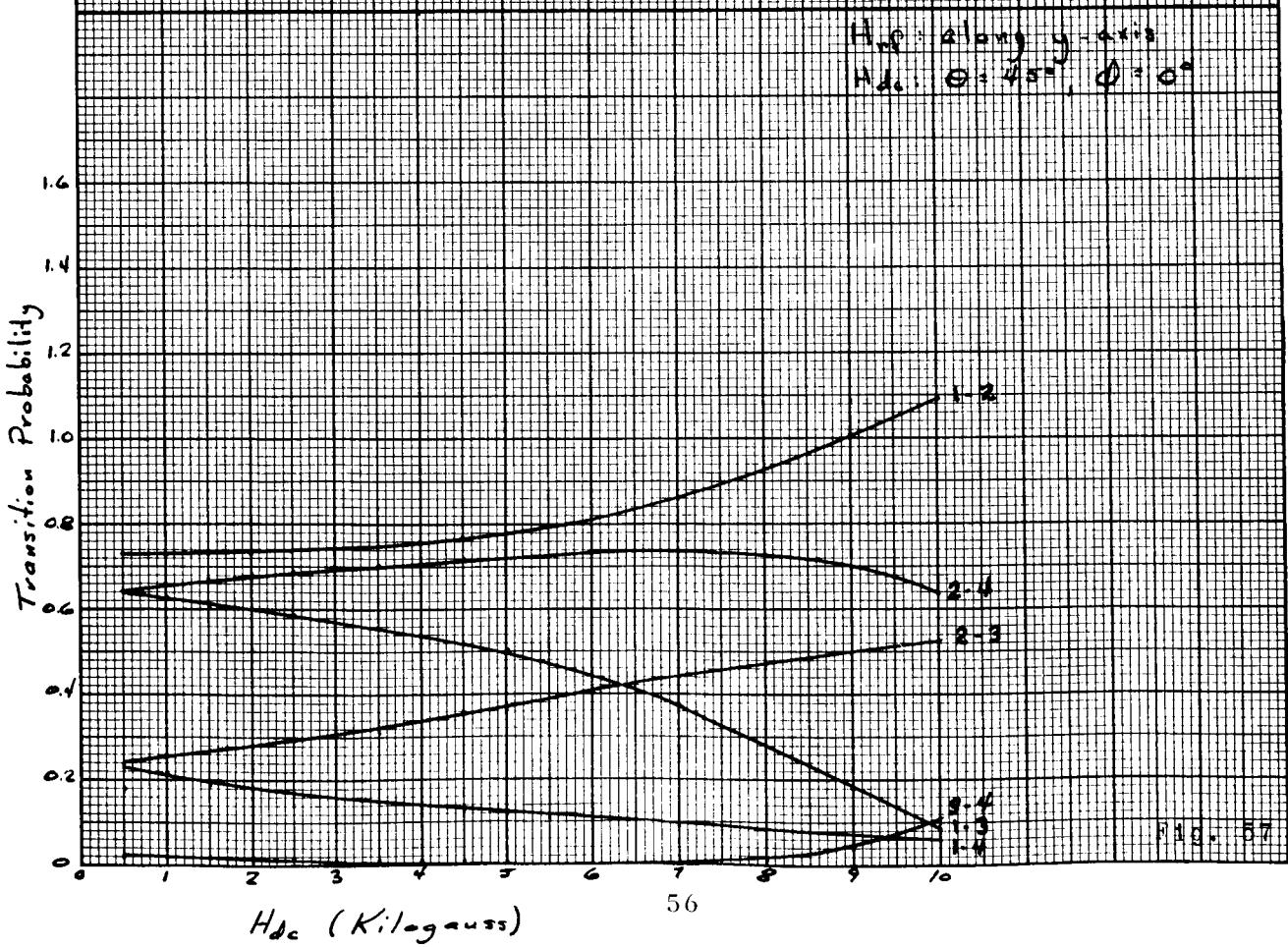


FIG. 57

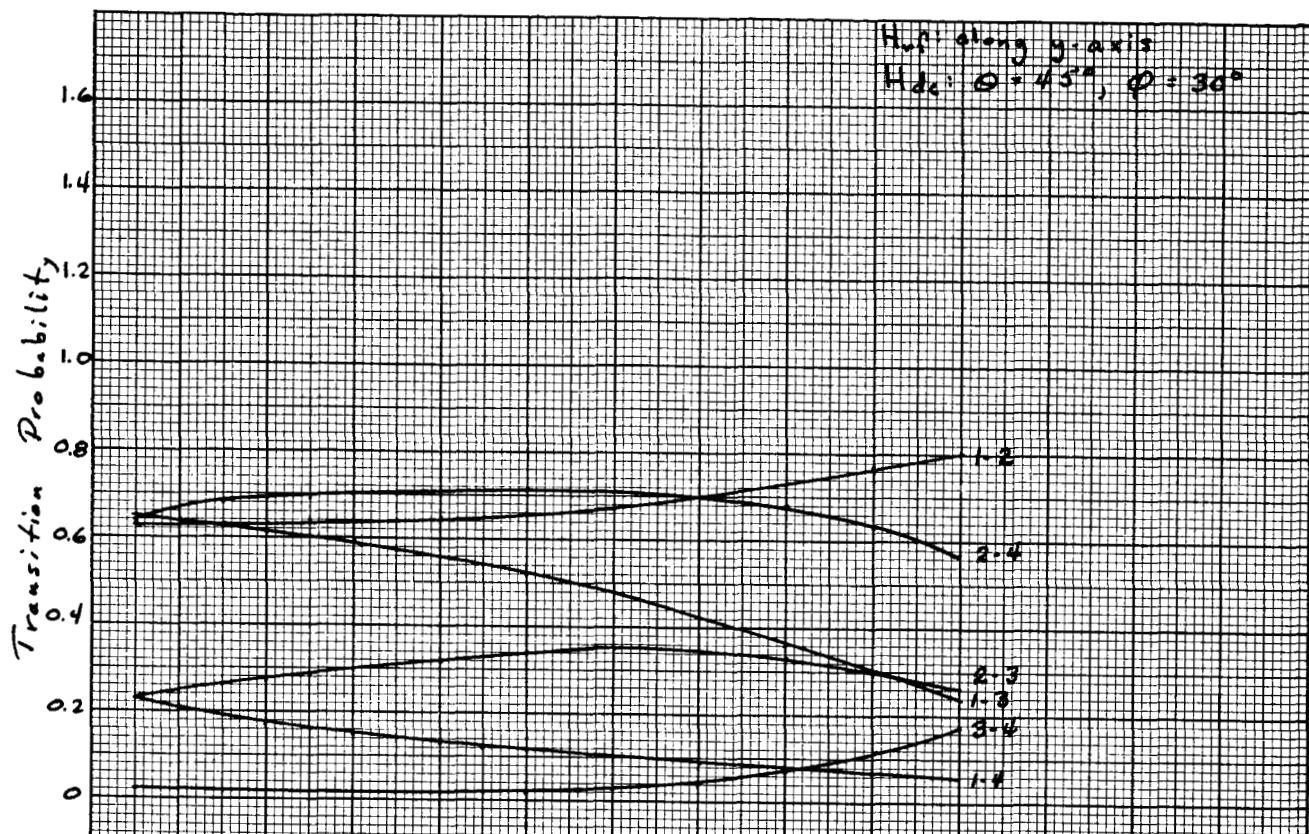


Fig. 58

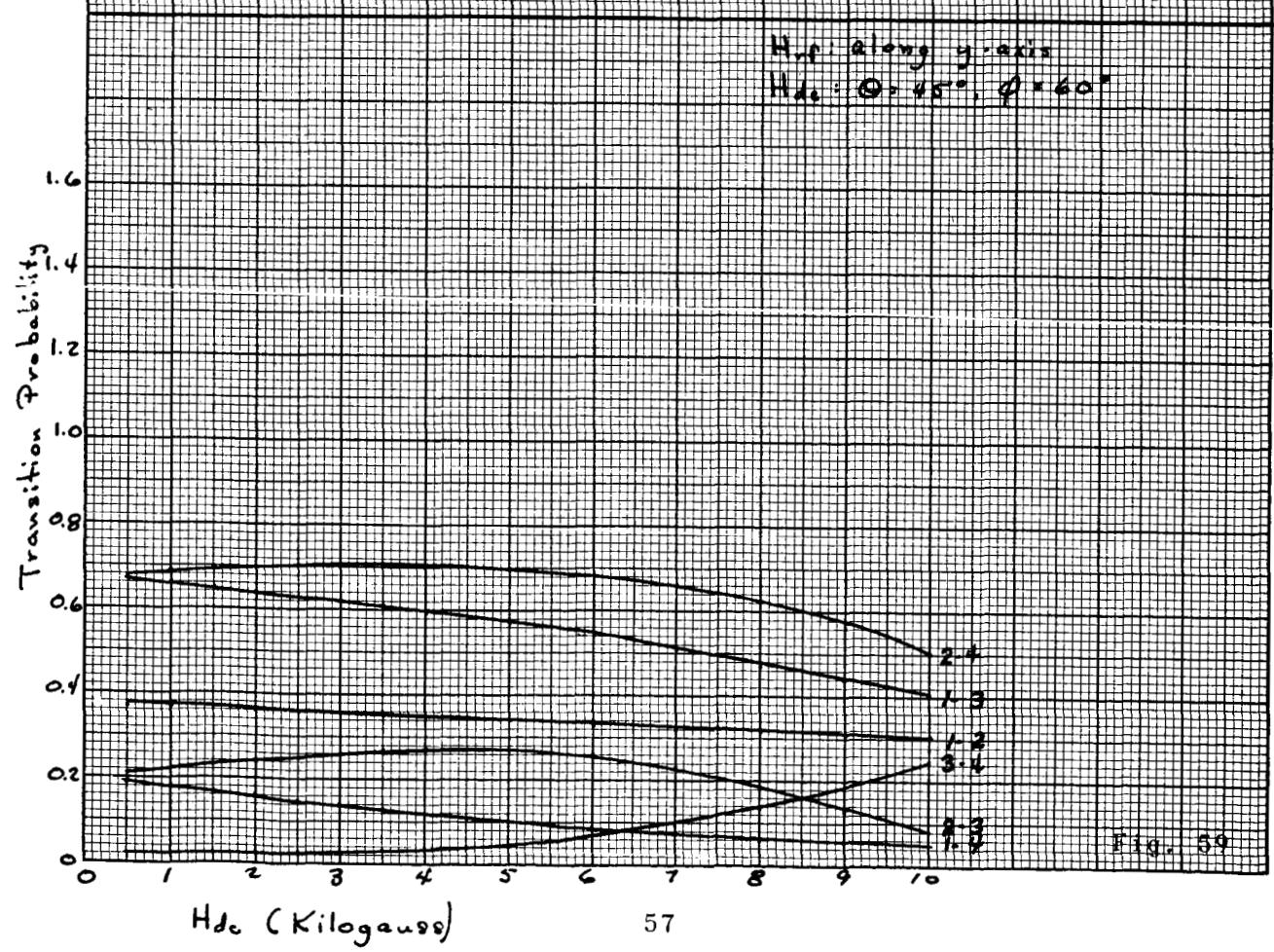
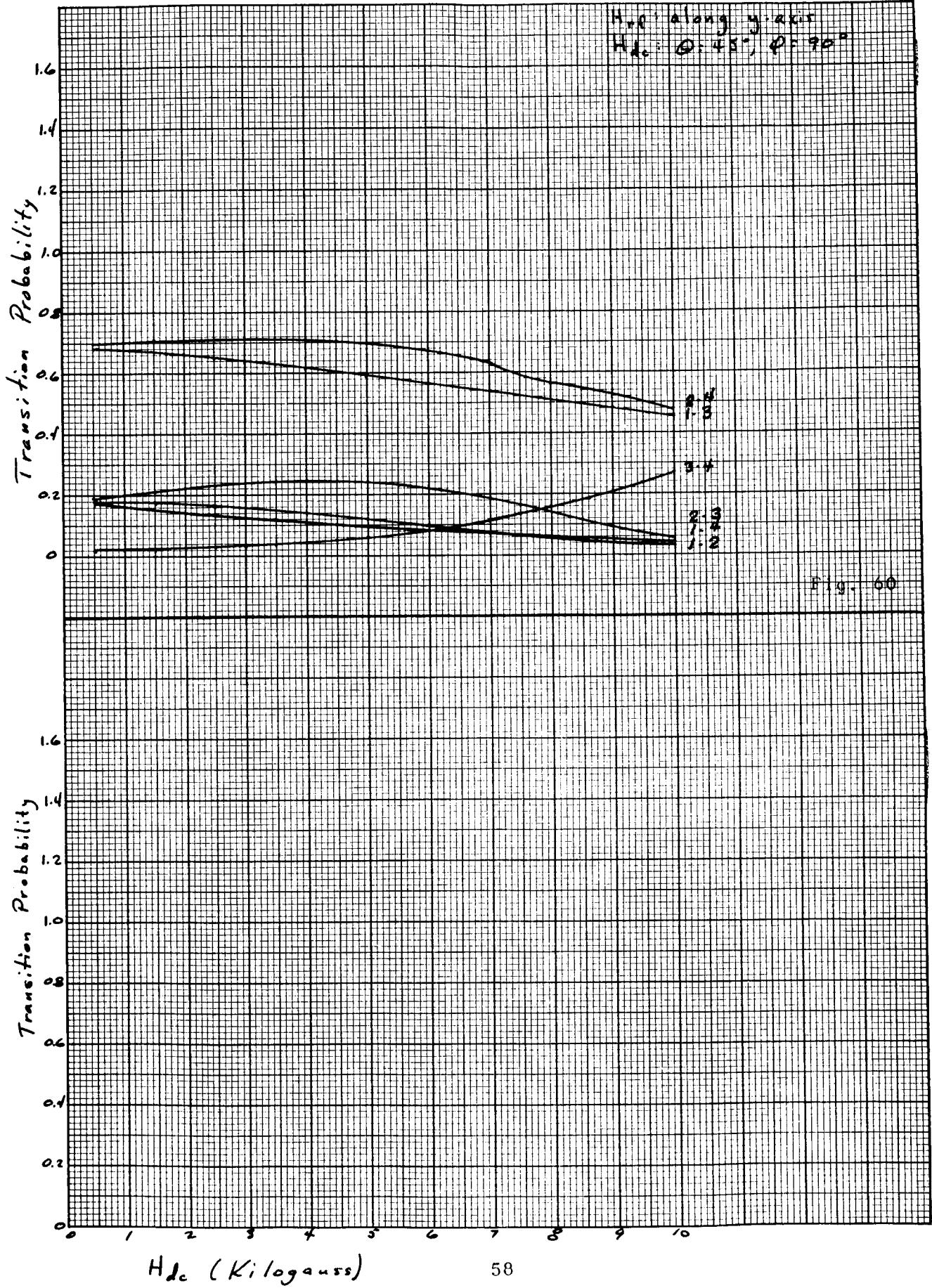
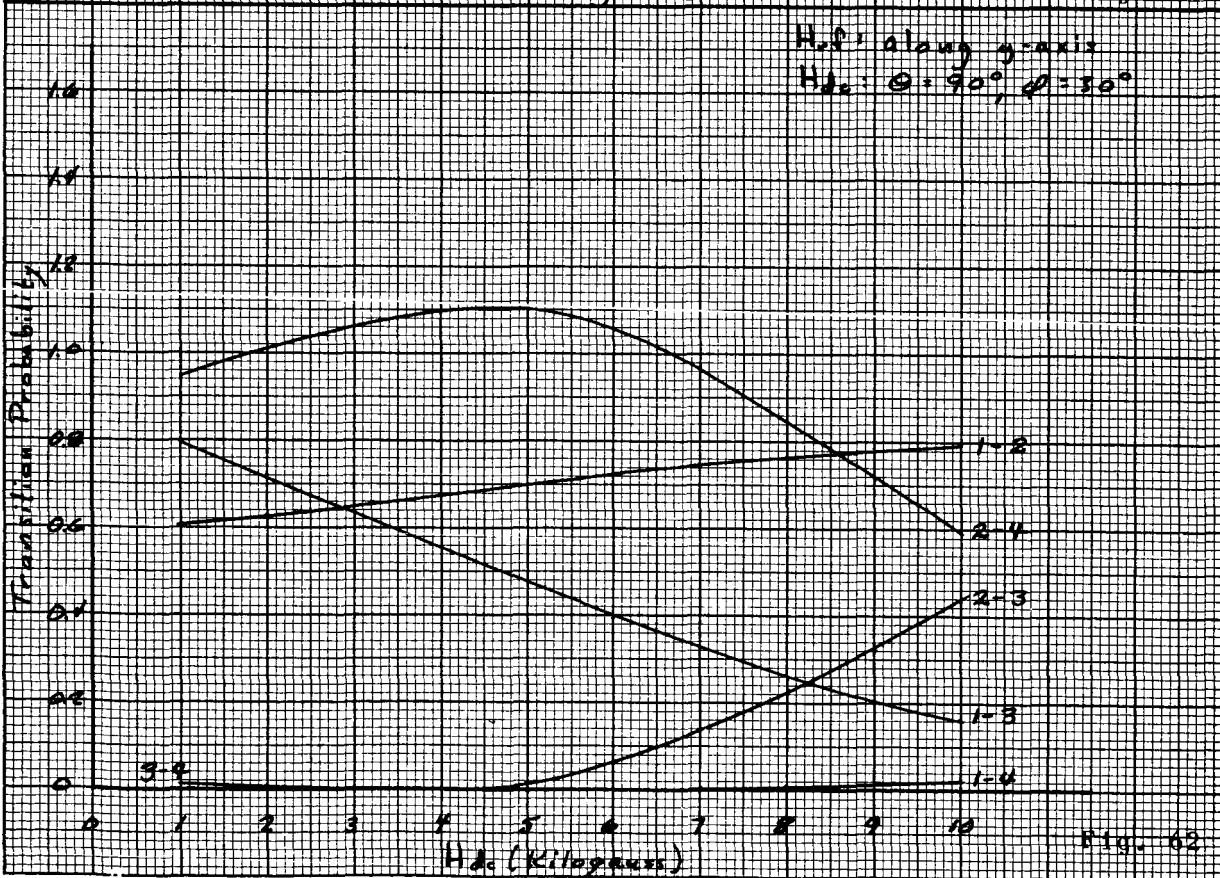
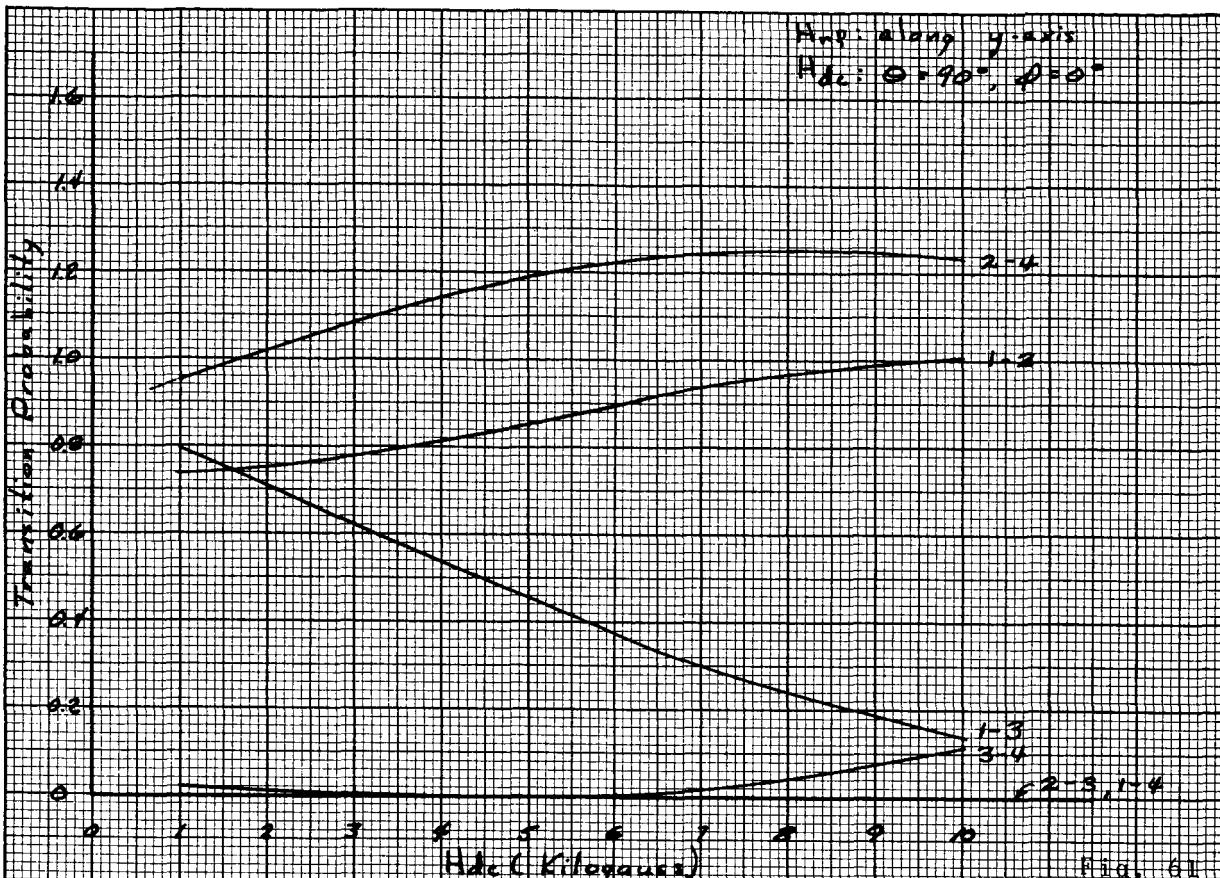
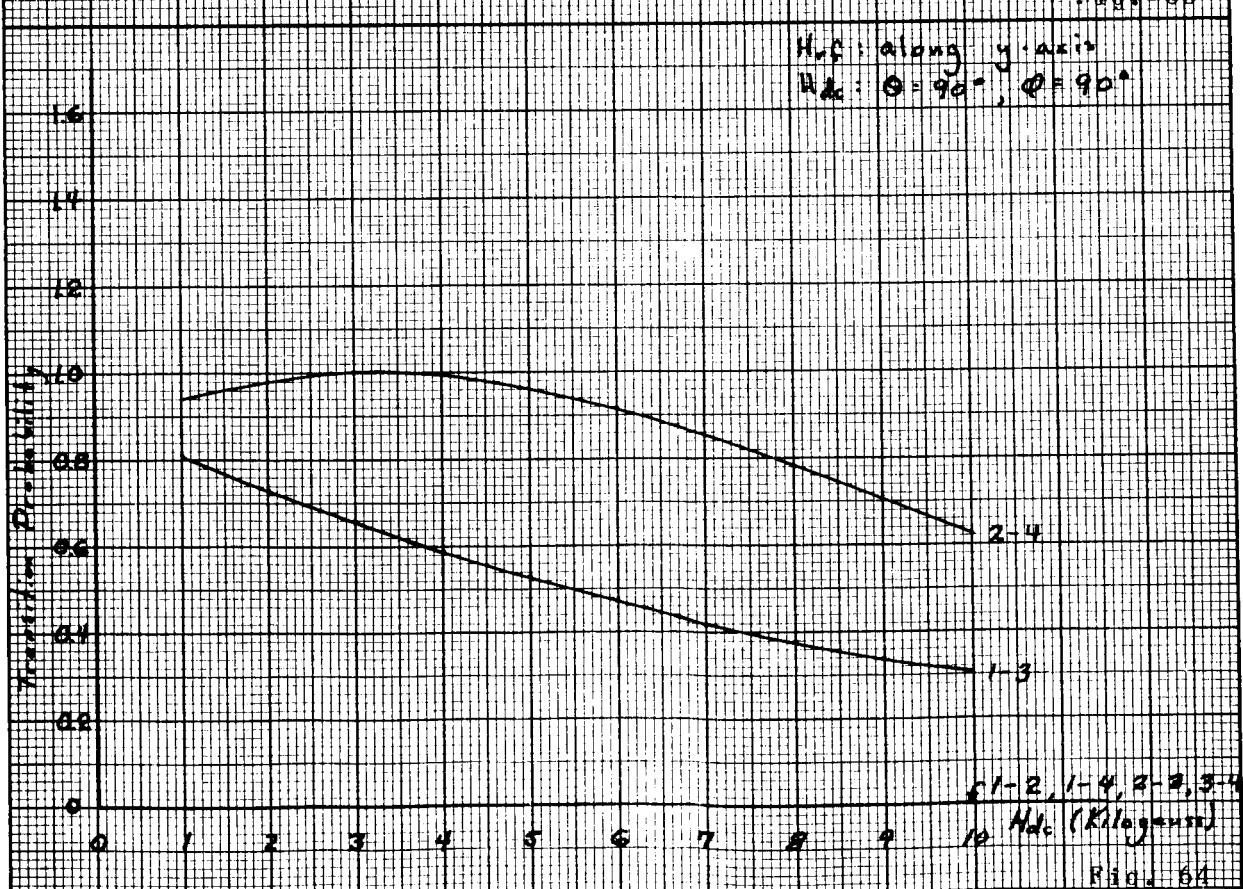
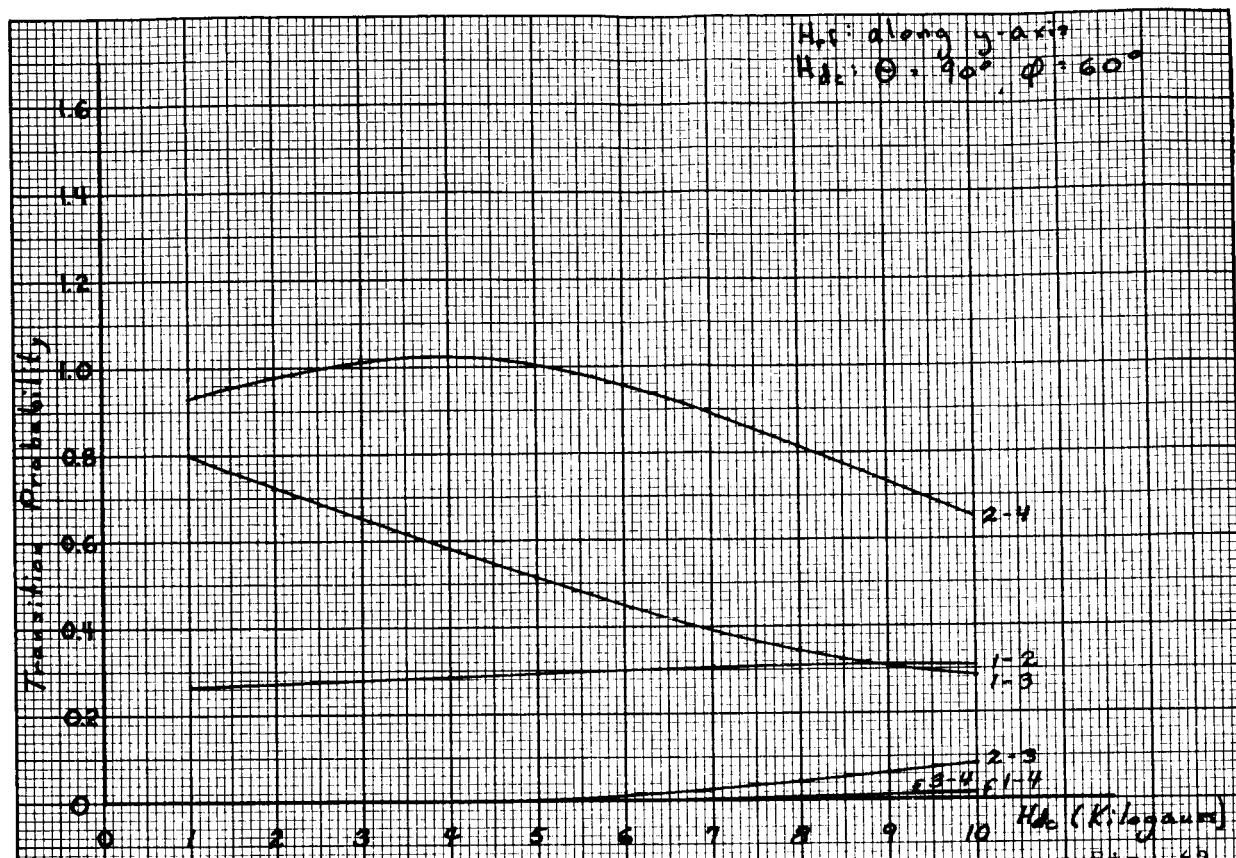


Fig. 59







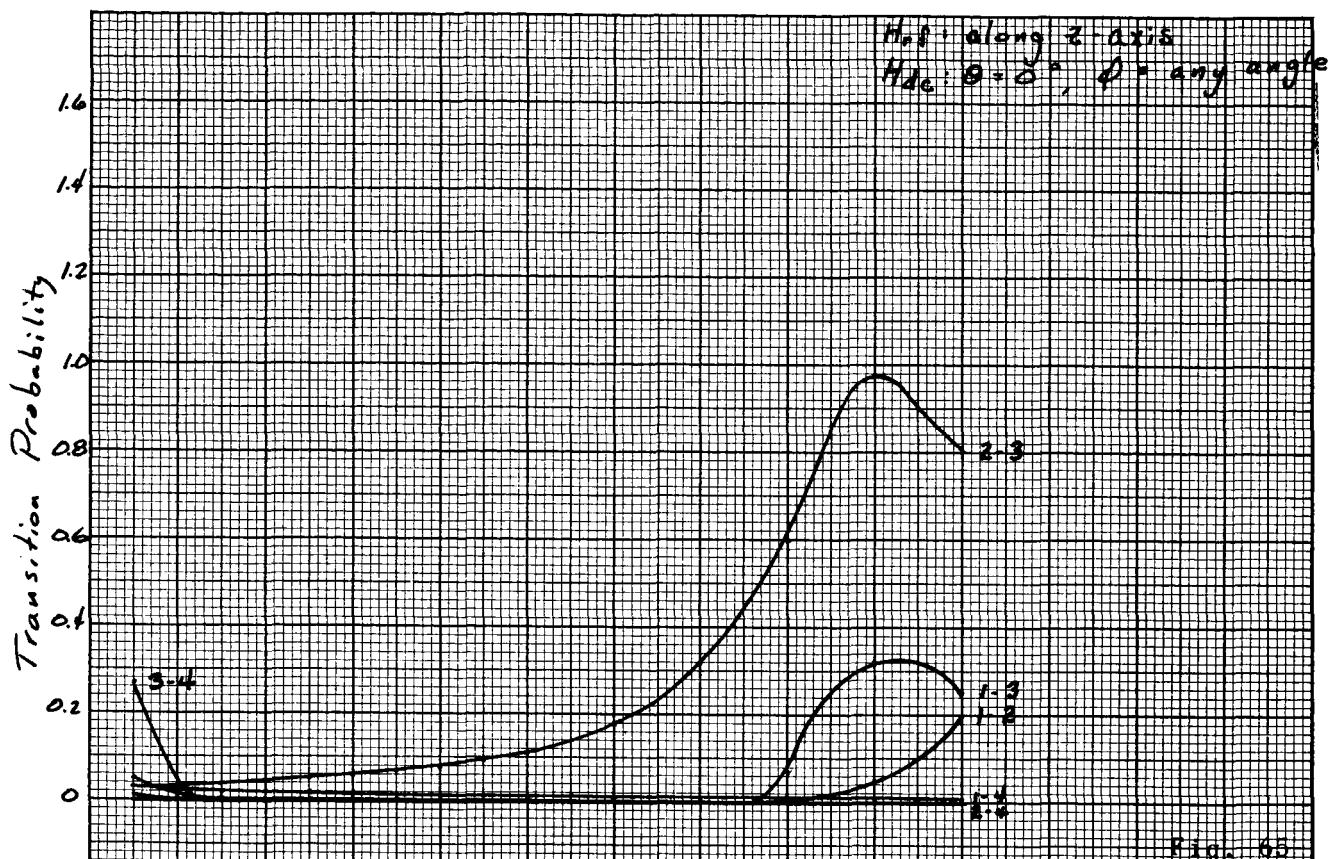


Fig. 65

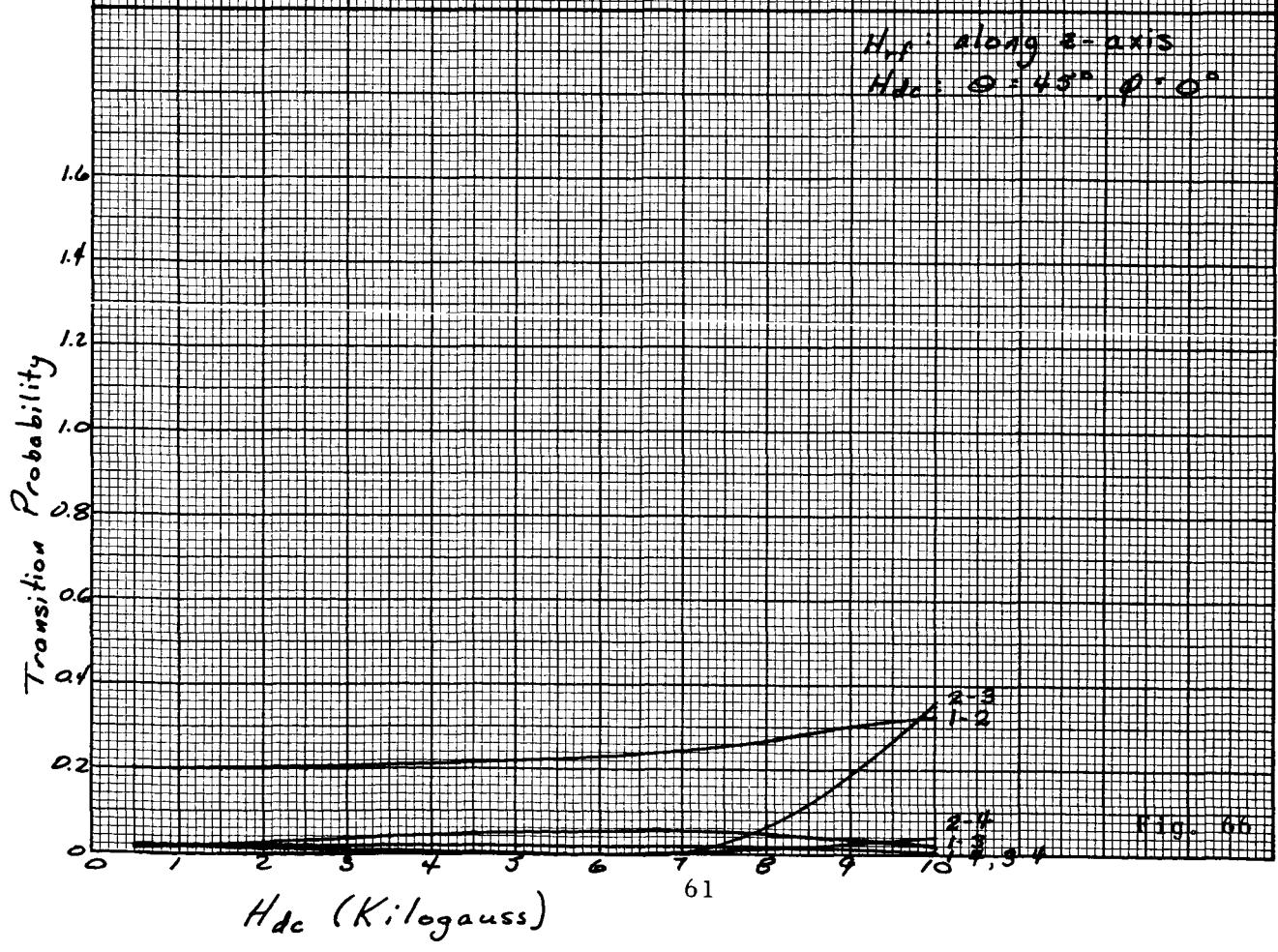


Fig. 66

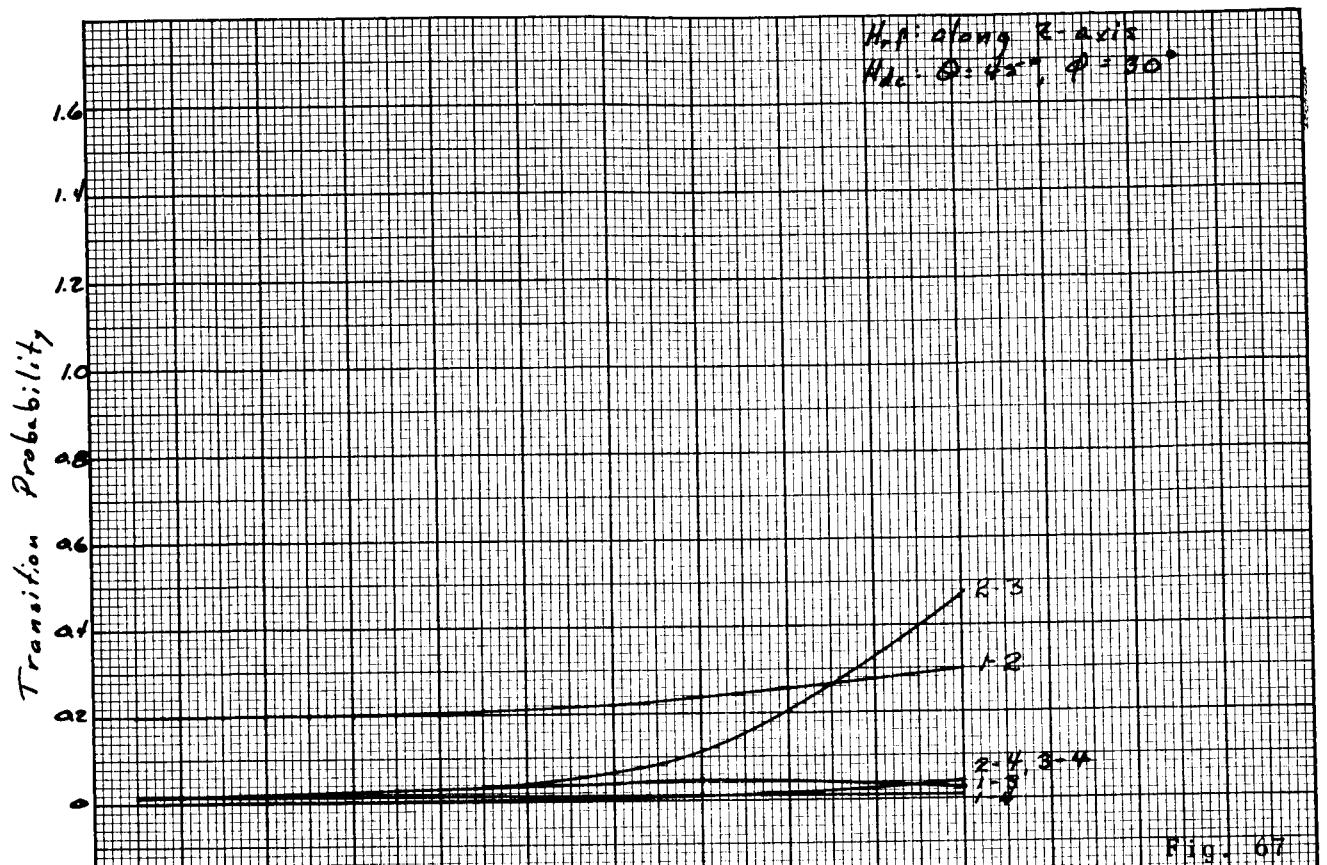


Fig. 67

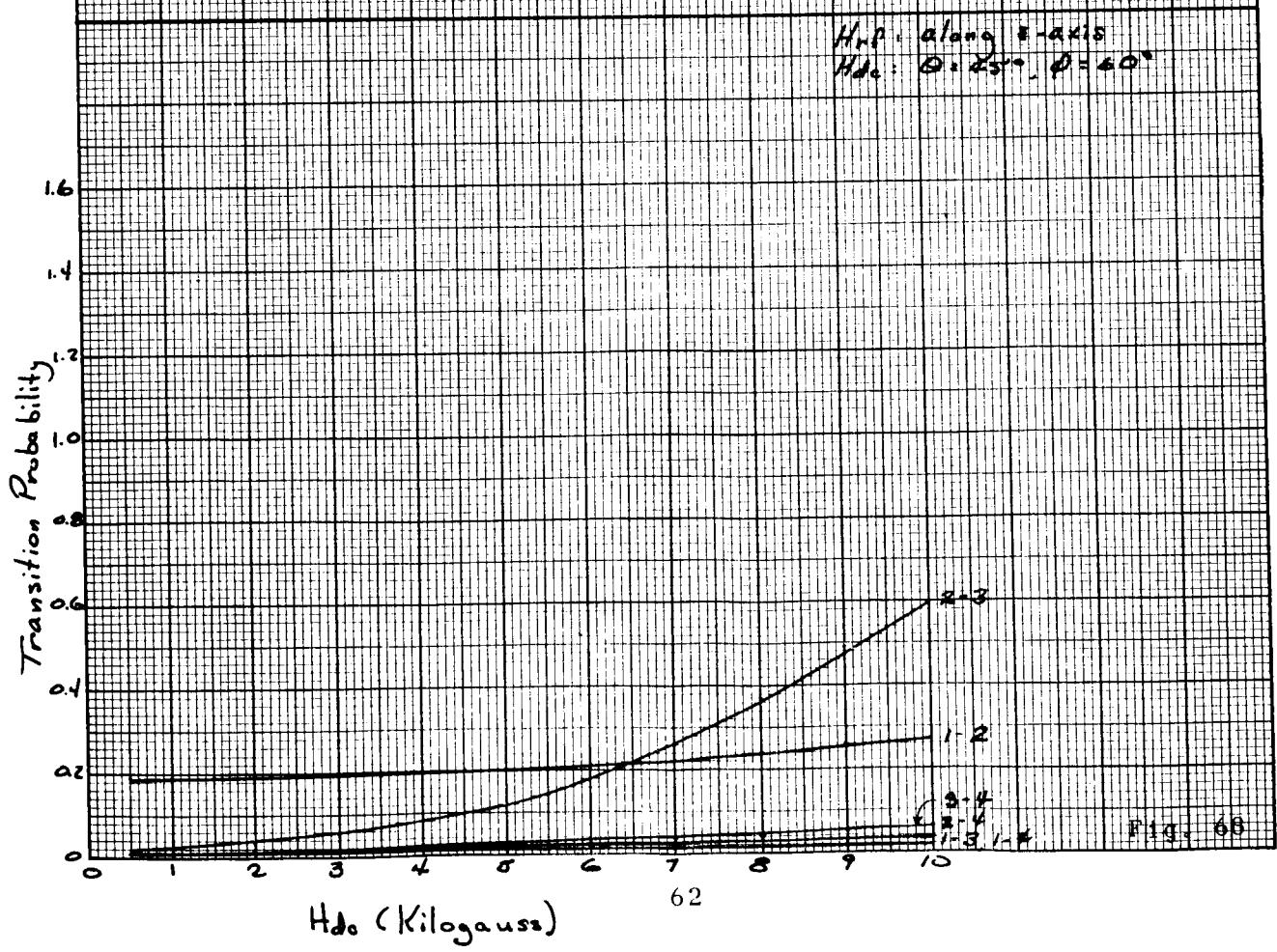
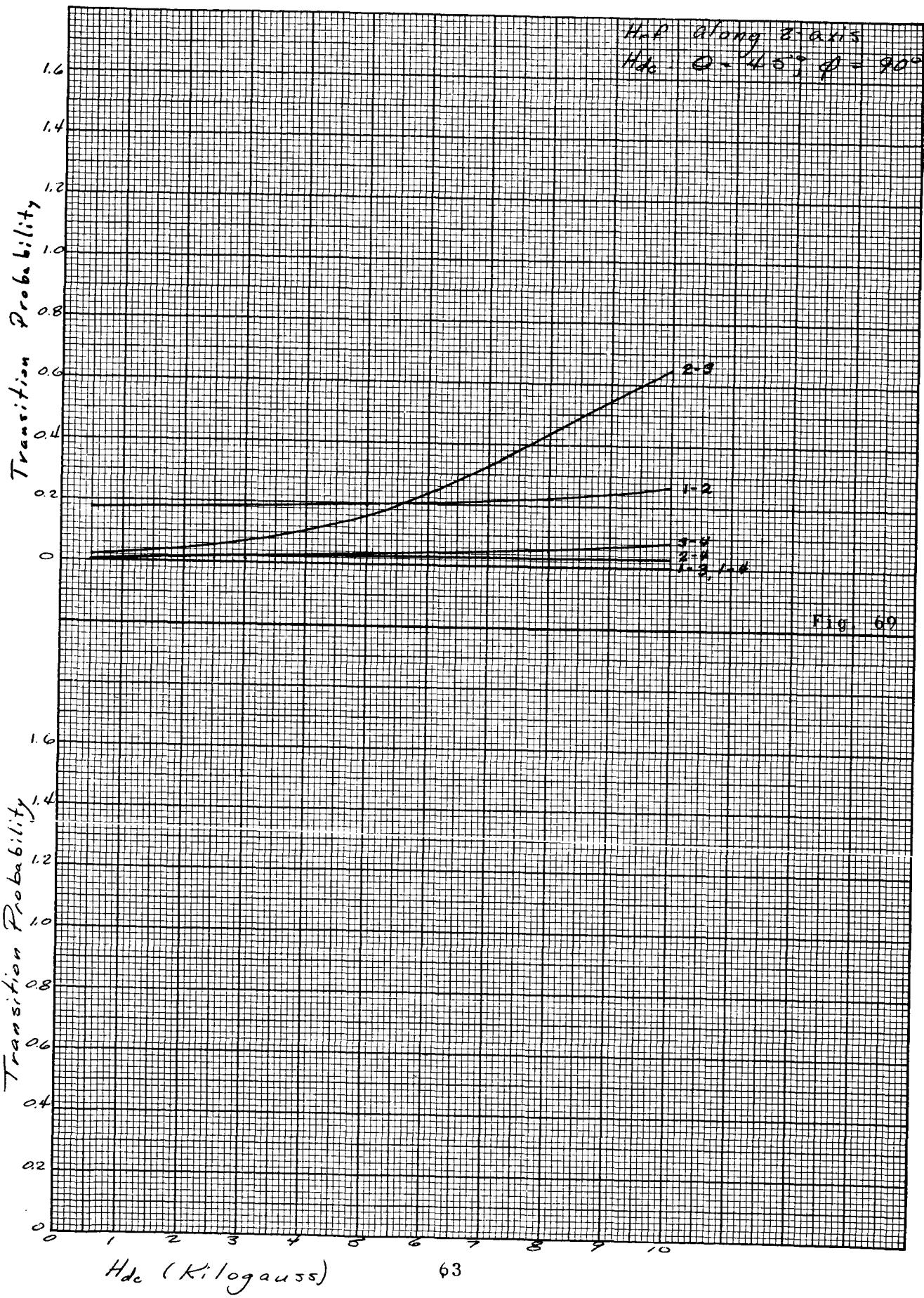
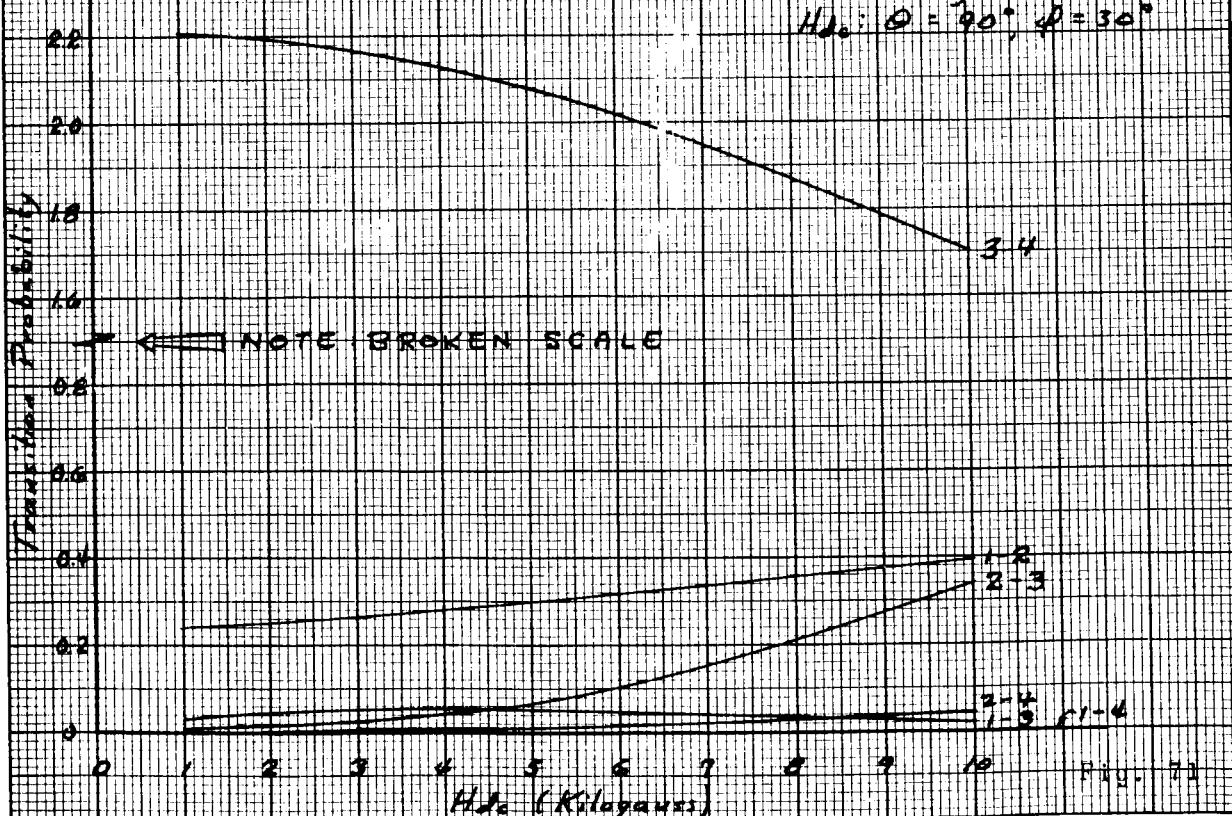
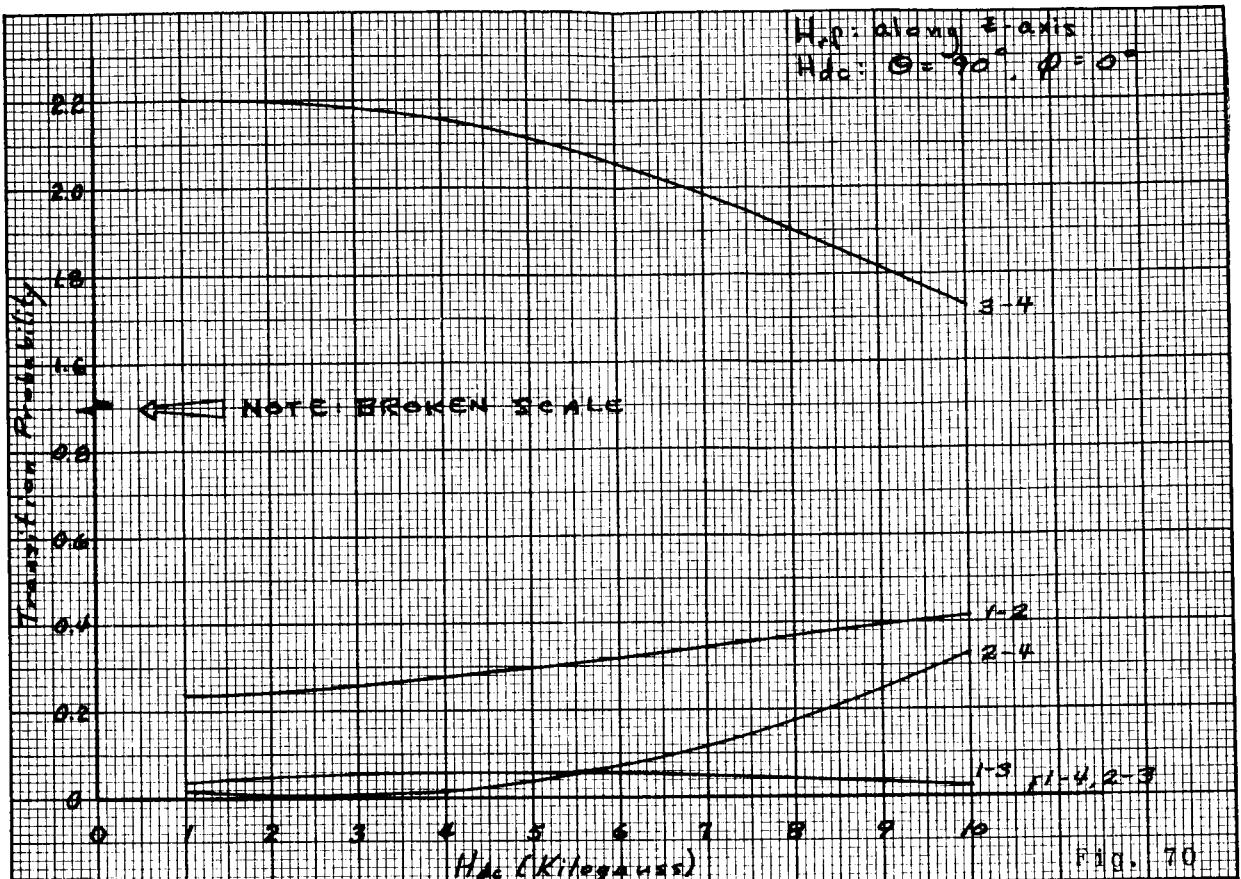
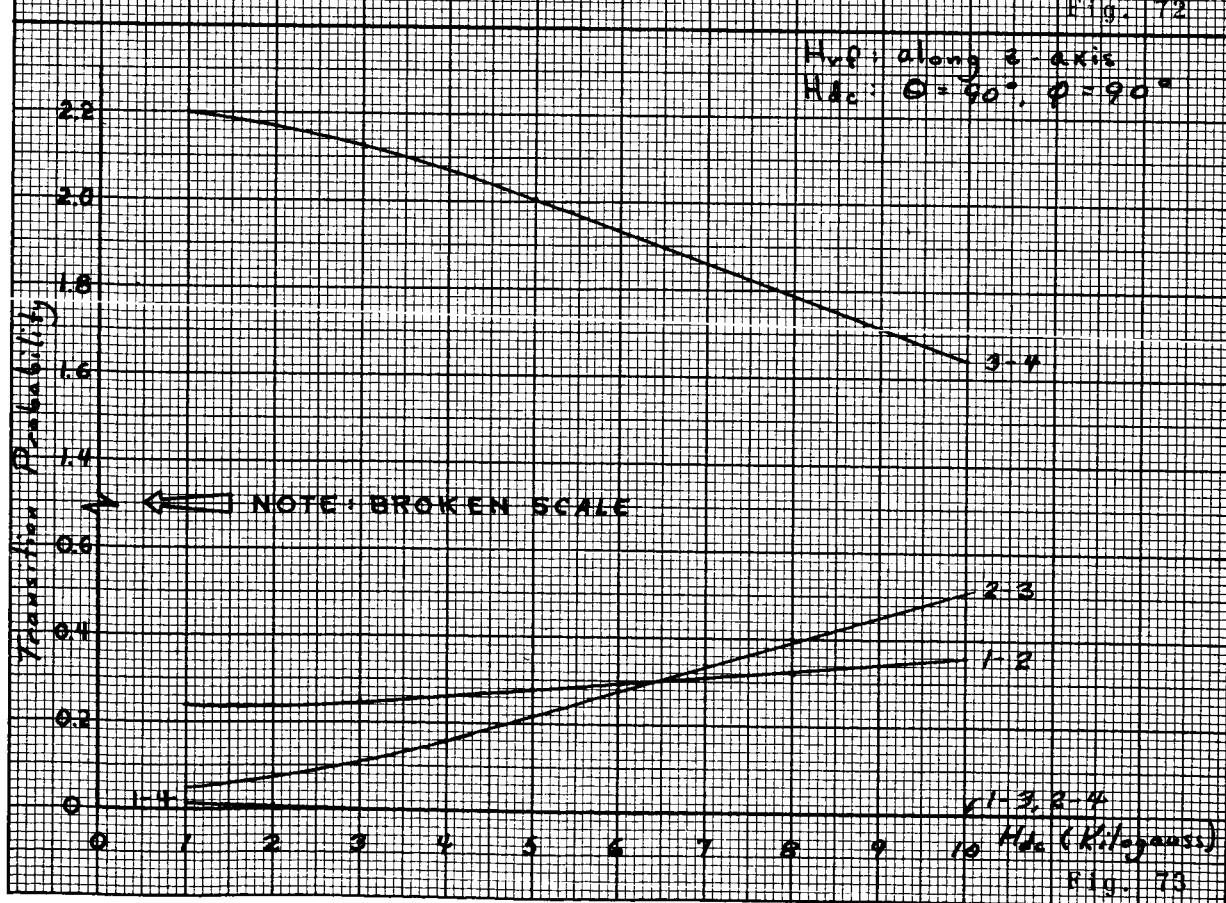
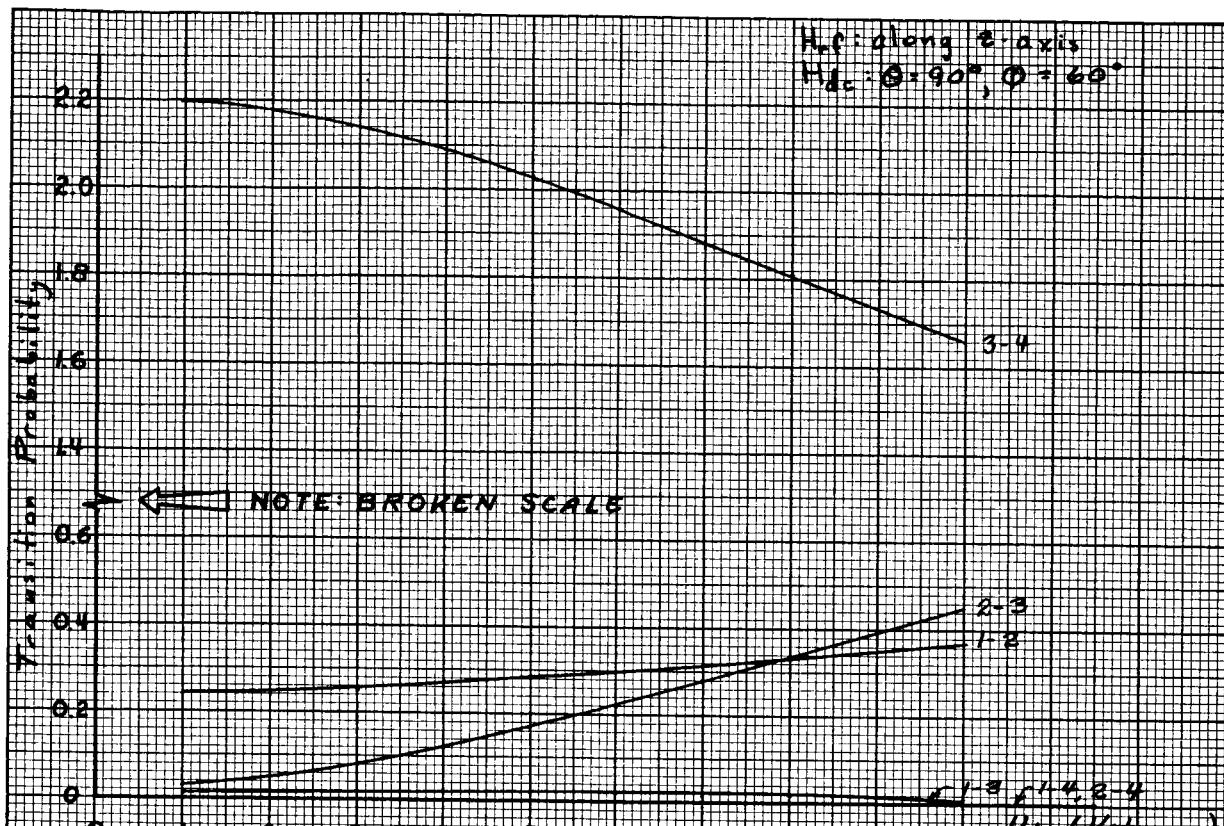


Fig. 68







Figs. 74-82

MAXIMUM TRANSITION PROBABILITIES FOR $Mn^{4+} : TiO_2$

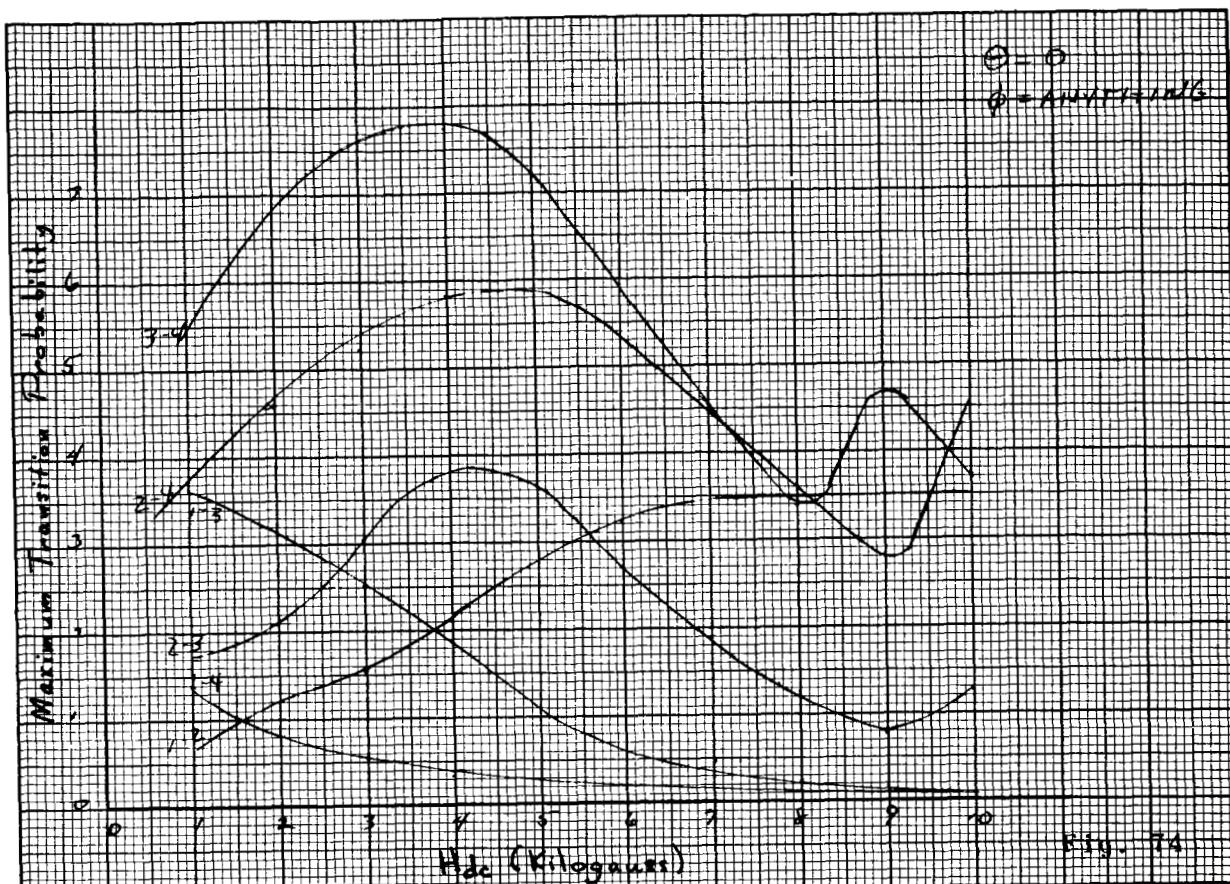


Fig. 74

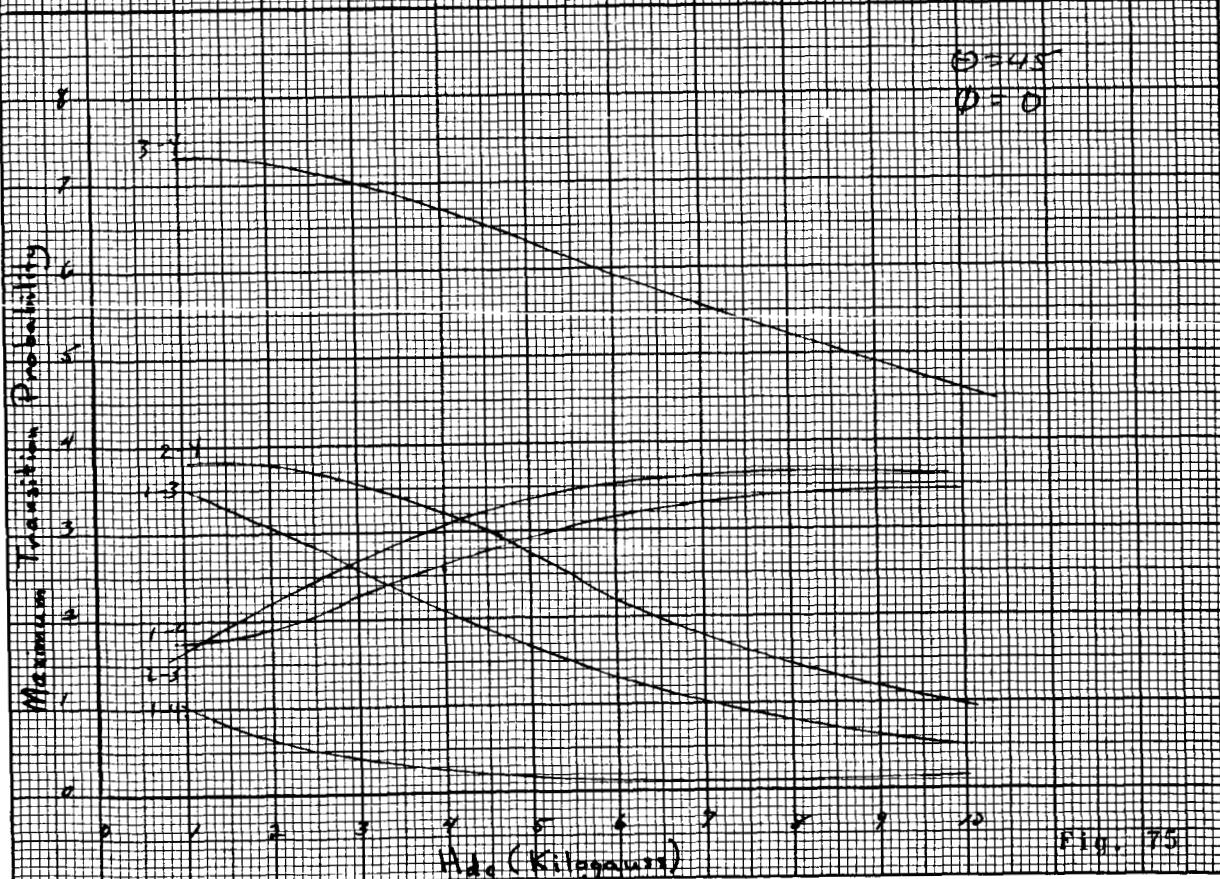
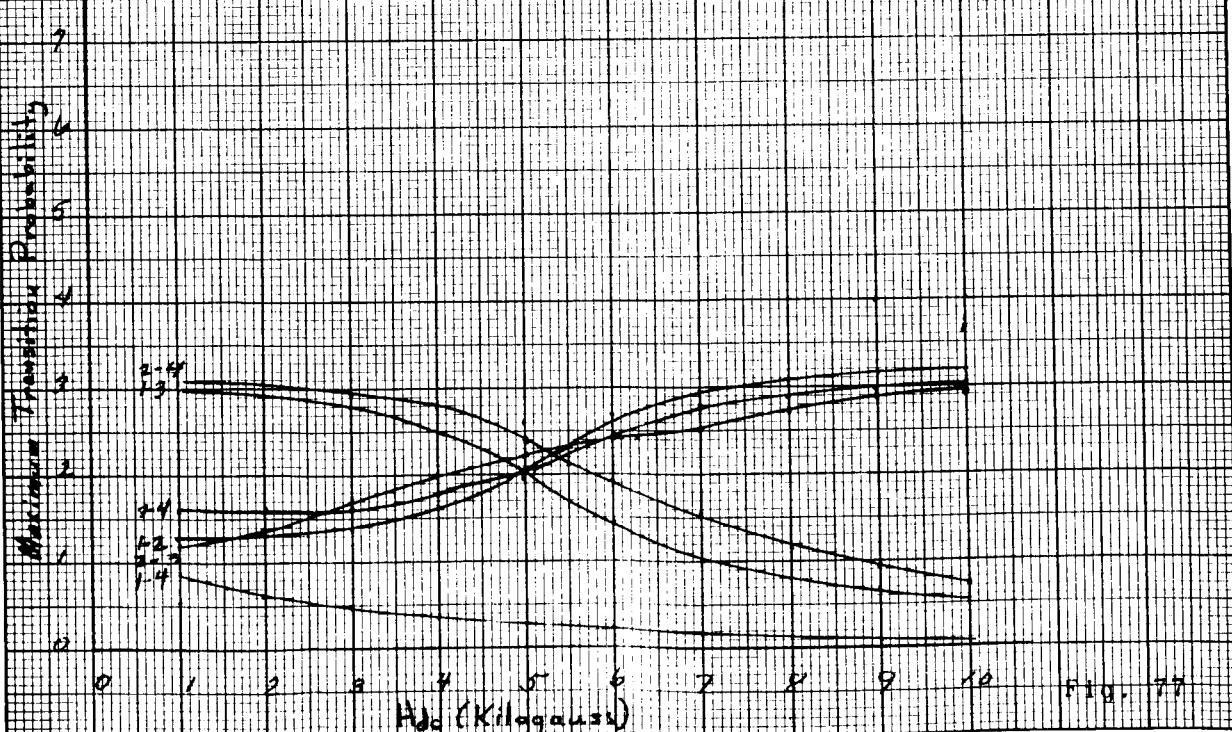
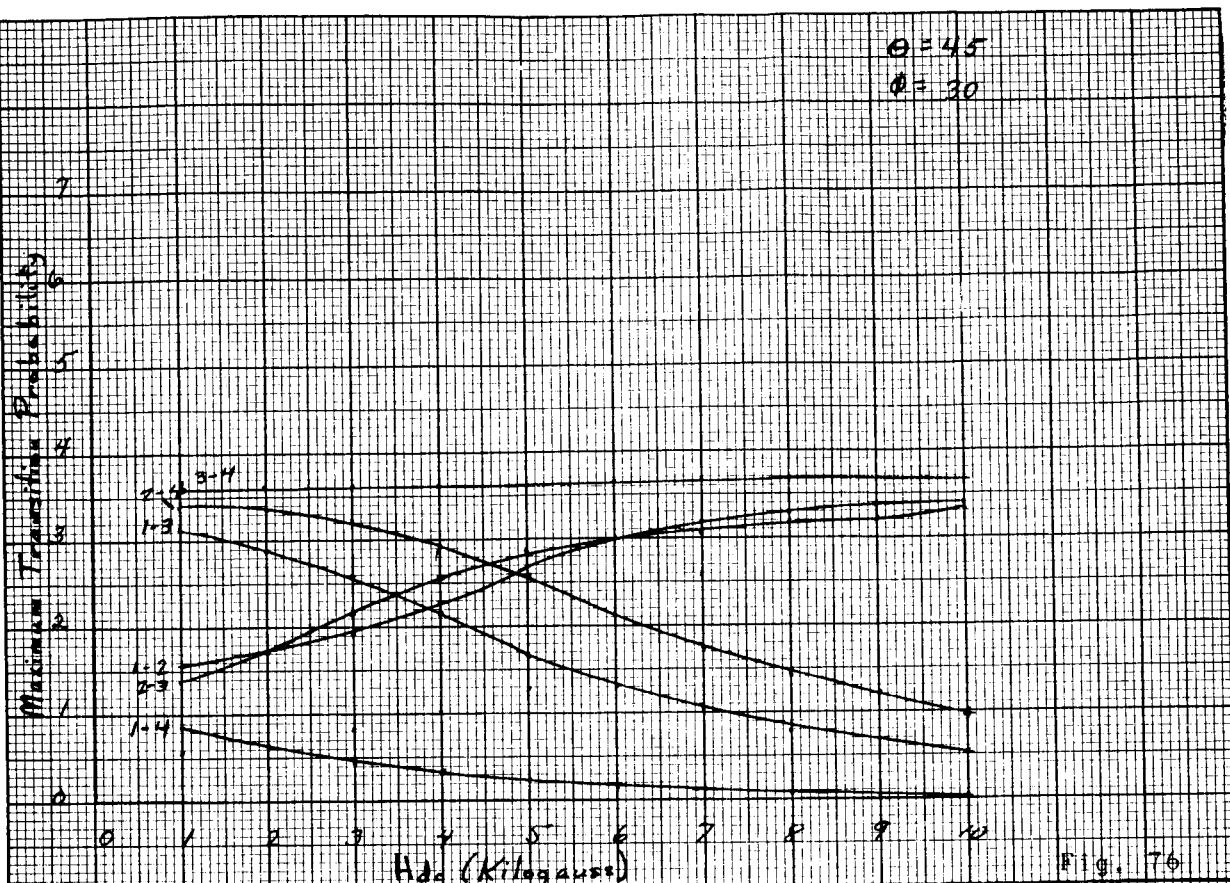
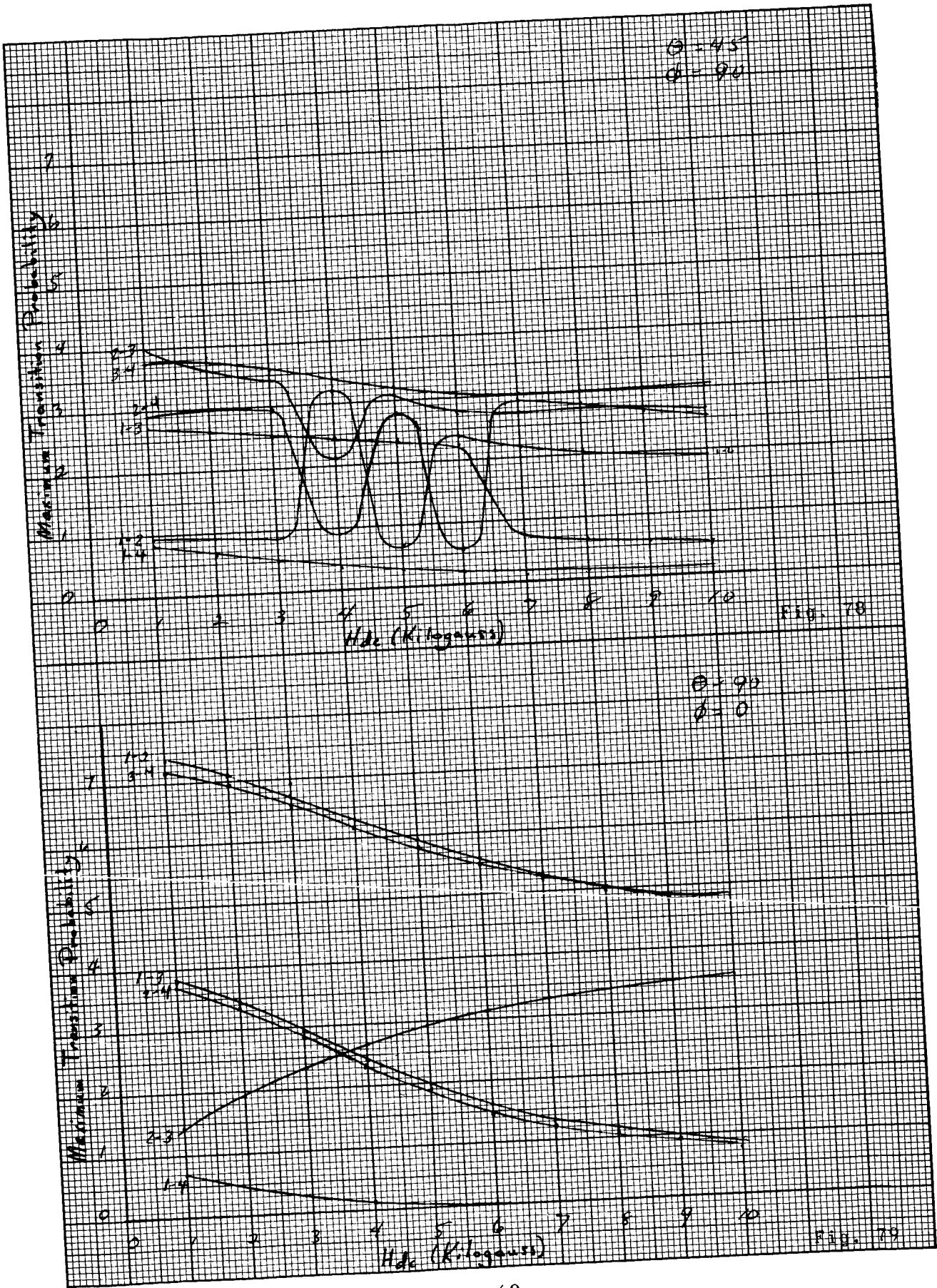


Fig. 75





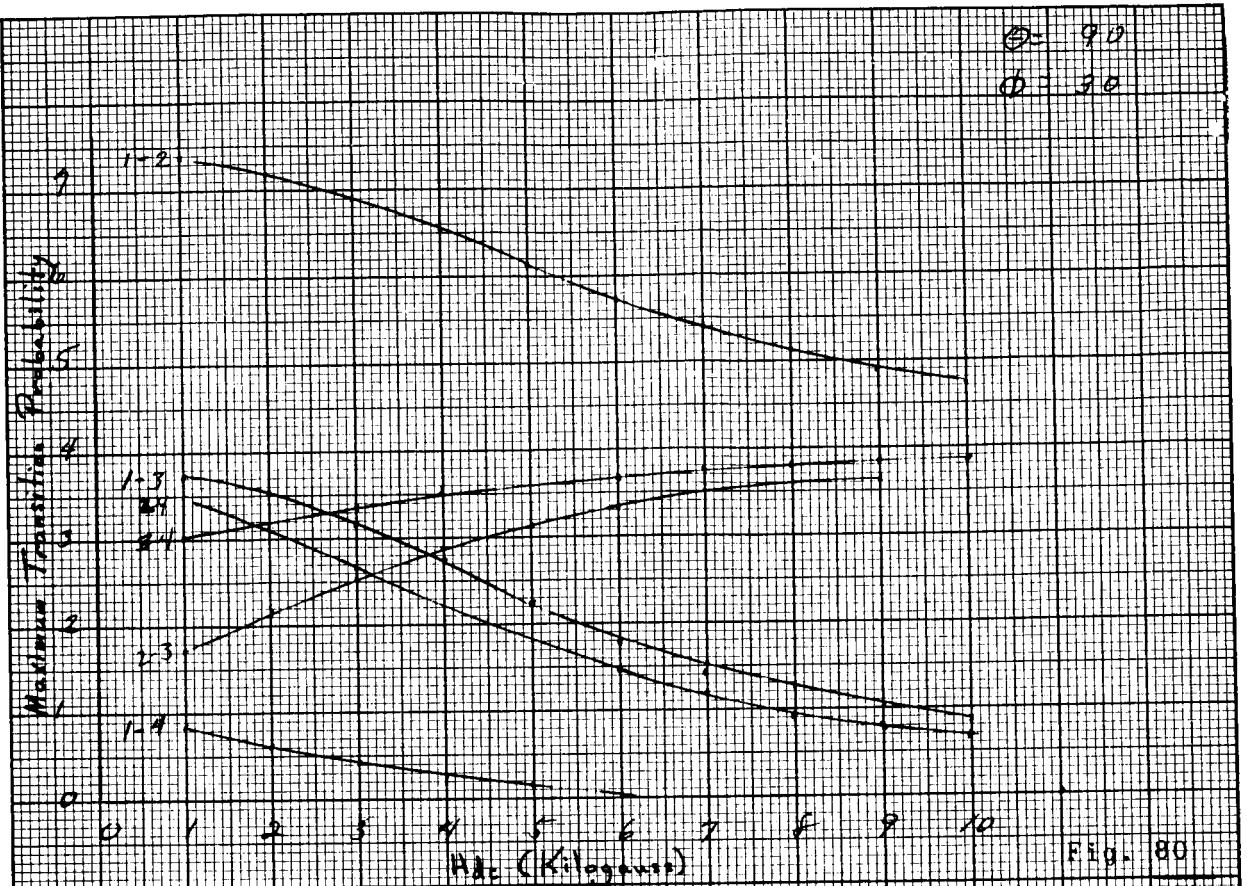


Fig. 80

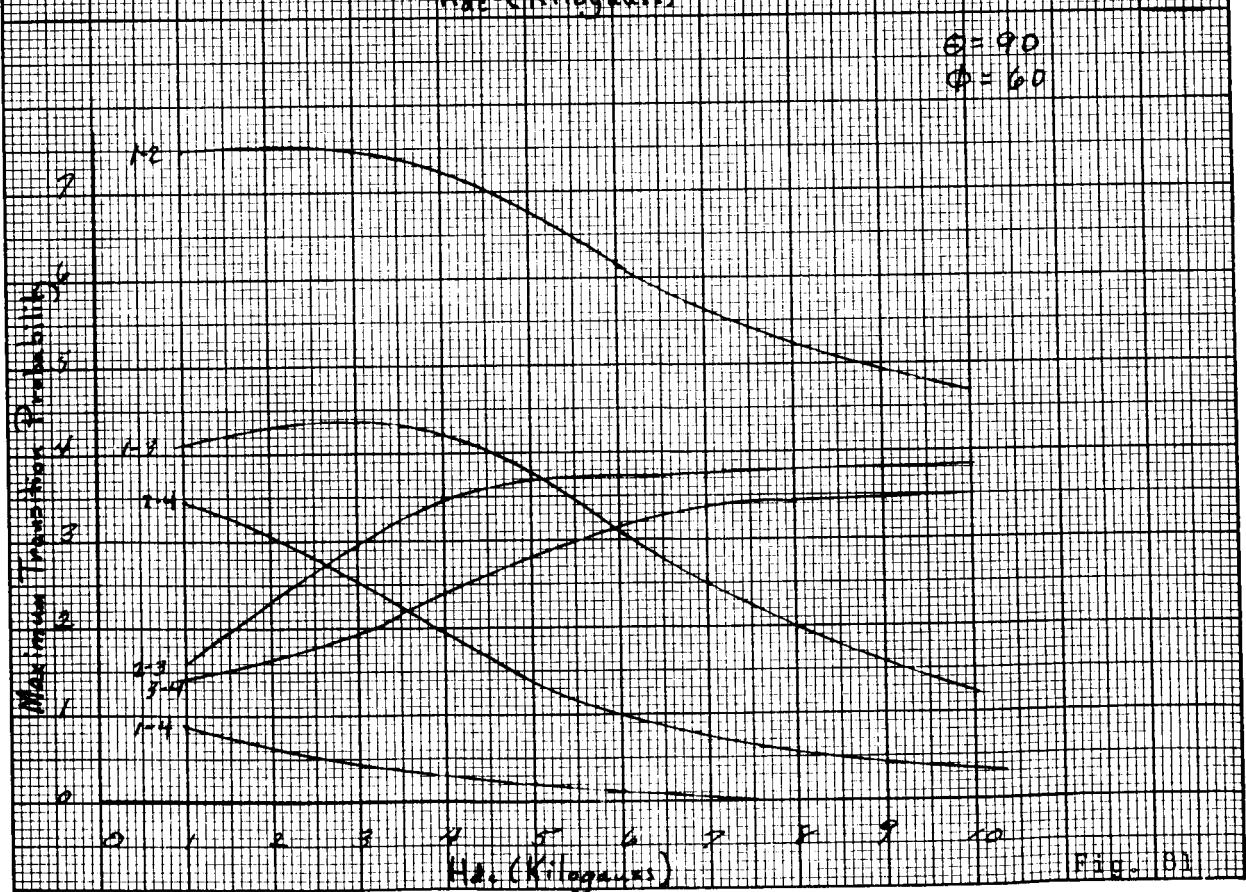
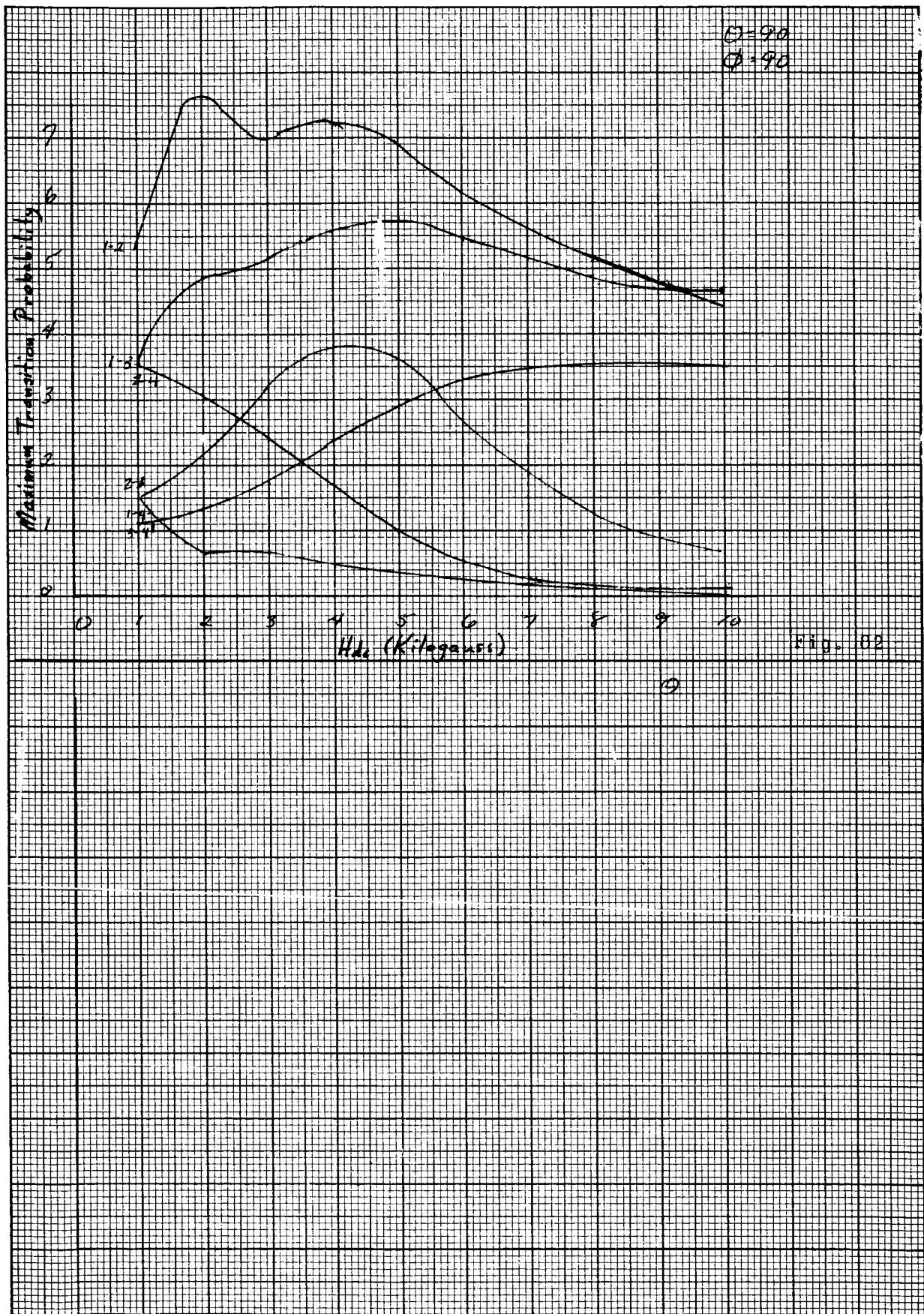


Fig. 81



Figs. 83-91

MAXIMUM TRANSITION PROBABILITIES FOR Cr³⁺:ZnWO₄

